> *restart*;

| **Правило** | **Альтернативные имена** | **Описание** |
|---|---|---|
| Постоянная | constant | $\int c\, dx = c\, x$ |
| Умножение на константу | constantmultiple | $\int c\, f(x)\, dx = c\left(\int f(x)\, dx\right)$ |
| правило дифференцирования | diff | $\int \frac{d}{dx} f(x)\, dx = f(x)$ |
| Разность | difference | $\int (f(x) - g(x))\, dx = \int f(x)\, dx - \left(\int g(x)\, dx\right)$ |
| Эквивалентность | identity | $\int x\, dx = \frac{1}{2}\, x^2$ |
| Частичные суммы | partialfractions | $\int f(x)\, dx = \int (R_1(x) + R_2(x) + ...)\, dx$, где $R_1(x) + R_2(x) + ...$ представление функции $f(x)$ |
| Степенное правило | power | $\int x^n\, dx = \frac{x^{n+1}}{n+1}$ <br> $\int_a^b x^n\, dx = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$ |
| Переход к старым переменным | revert | Обратная замена переменных |
| Решение | solve | Алгебраически решают уравнение, в котором один и тот же интеграл появляется более одного раза (часто используется после нескольких этапов интегрирования по частям) |
| Сумма | sum | $\int (f(x) + g(x))\, dx = \int f(x)\, dx + \int g(x)\, dx$ |

> *with*(*Student:-Calculus1*) :

> *#infolevel[Student[Calculus1]]*:= 1 :

> $J_1 := \int \sin(3\,x)\sin(5\,x)\, dx : J_2 := Int\left(\sin\left(3\,x - \frac{\pi}{4}\right)\cos\left(x + \frac{\pi}{4}\right), x\right) : J_3 := Int\left(\sin^3(x), x\right) :$
$J_4 := Int\left(\cos^2(x)\sin^3(x), x\right) :$

> $seq(I_j = J_j, j = 1..4)$

$$I_1 = \int \sin(3\,x)\,\sin(5\,x)\, dx,\ I_2 = \int \left(-\cos\left(3\,x + \frac{1}{4}\,\pi\right)\cos\left(x + \frac{1}{4}\,\pi\right)\right) dx,\ I_3 = \int \sin(x)^3\, dx,\ I_4 = \int \cos(x)^2\,\sin(x)^3\, dx \qquad \textbf{(1)}$$

> $\mathrm{mess}_1$ := *sprintf*("стр.: %d, Высшая математика, %d. Киреева ЮГ, Петров ВВ", 51, 4) :

> $seq(Hint(J_i), i = 1..4)$ :

>

> *ShowSolution*(GetProblem(1, internal)); $\text{mess}_1$;

$$\int \sin(3\,x)\,\sin(5\,x)\,\mathrm{d}x$$

$$= \int \left(\frac{\cos(2\,x)}{2} - \frac{\cos(8\,x)}{2}\right) \mathrm{d}x \qquad \left[\mathit{rewrite},\ \sin(3\,x)\,\sin(5\,x) = \frac{\cos(2\,x)}{2} - \frac{\cos(8\,x)}{2}\right]$$

$$= \int \frac{\cos(2\,x)}{2}\,\mathrm{d}x + \int -\frac{\cos(8\,x)}{2}\,\mathrm{d}x \qquad [\mathit{sum}]$$

$$= \frac{\left(\int \cos(2\,x)\,\mathrm{d}x\right)}{2} + \int -\frac{\cos(8\,x)}{2}\,\mathrm{d}x \qquad [\mathit{constantmultiple}]$$

$$= \frac{\left(\int \frac{\cos(u)}{2}\,\mathrm{d}u\right)}{2} + \int -\frac{\cos(8\,x)}{2}\,\mathrm{d}x \qquad [\mathit{change},\ u = 2\,x,\ u]$$

$$= \frac{\left(\int \cos(u)\,\mathrm{d}u\right)}{4} + \int -\frac{\cos(8\,x)}{2}\,\mathrm{d}x \qquad [\mathit{constantmultiple}]$$

$$= \frac{\sin(u)}{4} + \int -\frac{\cos(8\,x)}{2}\,\mathrm{d}x \qquad [\cos]$$

$$= \frac{\sin(2\,x)}{4} + \int -\frac{\cos(8\,x)}{2}\,\mathrm{d}x \qquad [\mathit{revert}]$$

$$= \frac{\sin(2\,x)}{4} - \frac{\left(\int \cos(8\,x)\,\mathrm{d}x\right)}{2} \qquad [\mathit{constantmultiple}]$$

$$= \frac{\sin(2\,x)}{4} - \frac{\left(\int \frac{\cos(u)}{8}\,\mathrm{d}u\right)}{2} \qquad [\mathit{change},\ u = 8\,x,\ u]$$

$$= \frac{\sin(2\,x)}{4} - \frac{\left(\int \cos(u)\,\mathrm{d}u\right)}{16} \qquad [\mathit{constantmultiple}]$$

$$= \frac{\sin(2\,x)}{4} - \frac{\sin(u)}{16} \qquad [\cos]$$

$$= \frac{\sin(2\,x)}{4} - \frac{\sin(8\,x)}{16} \qquad [\mathit{revert}]$$

"стр.: 51, Высшая математика, 4. Киреева ЮГ, Петров ВВ" **(2)**

> *ShowSolution*(GetProblem(2, internal)); $\text{mess}_1$;

$$\int -\cos\left(3x+\frac{\pi}{4}\right)\cos\left(x+\frac{\pi}{4}\right)\,dx$$

$$= -\left(\int \cos\left(3x+\frac{\pi}{4}\right)\cos\left(x+\frac{\pi}{4}\right)dx\right) \qquad [constantmultiple]$$

$$= -\left(\int\left(\frac{\cos(2x)}{2}-\frac{\sin(4x)}{2}\right)dx\right) \qquad \left[rewrite, \cos\left(3x+\frac{\pi}{4}\right)\cos\left(x+\frac{\pi}{4}\right)=\frac{\cos(2x)}{2}-\frac{\sin(4x)}{2}\right]$$

$$= -\left(\int\frac{\cos(2x)}{2}dx\right)-\left(\int -\frac{\sin(4x)}{2}dx\right) \qquad [sum]$$

$$= -\frac{\left(\int\cos(2x)\,dx\right)}{2}-\left(\int -\frac{\sin(4x)}{2}dx\right) \qquad [constantmultiple]$$

$$= -\frac{\left(\int\frac{\cos(u)}{2}du\right)}{2}-\left(\int -\frac{\sin(4x)}{2}dx\right) \qquad [change, u=2x, u]$$

$$= -\frac{\left(\int\cos(u)\,du\right)}{4}-\left(\int -\frac{\sin(4x)}{2}dx\right) \qquad [constantmultiple]$$

$$= -\frac{\sin(u)}{4}-\left(\int -\frac{\sin(4x)}{2}dx\right) \qquad [\cos]$$

$$= -\frac{\sin(2x)}{4}-\left(\int -\frac{\sin(4x)}{2}dx\right) \qquad [revert]$$

$$= -\frac{\sin(2x)}{4}+\frac{\left(\int\sin(4x)\,dx\right)}{2} \qquad [constantmultiple]$$

$$= -\frac{\sin(2x)}{4}+\frac{\left(\int\frac{\sin(u)}{4}du\right)}{2} \qquad [change, u=4x, u]$$

$$= -\frac{\sin(2x)}{4}+\frac{\left(\int\sin(u)\,du\right)}{8} \qquad [constantmultiple]$$

$$= -\frac{\sin(2x)}{4}-\frac{\cos(u)}{8} \qquad [\sin]$$

$$= -\frac{\sin(2x)}{4}-\frac{\cos(4x)}{8} \qquad [revert]$$

"стр.: 51, Высшая математика, 4. Киреева ЮГ, Петров ВВ" **(3)**

> *ShowSolution*(GetProblem(3, internal)); $mess_1$;

$$\int \sin(x)^3\,dx$$

| | |
|---|---|
| $= \int (1-\cos(x)^2)\sin(x)\,dx$ | [*rewrite*, $\sin(x)^2 = 1-\cos(x)^2$] |
| $= \int (\sin(x)-\sin(x)\cos(x)^2)\,dx$ | [*rewrite*, $(1-\cos(x)^2)\sin(x) = \sin(x)-\sin(x)\cos(x)^2$] |
| $= \int \sin(x)\,dx + \int -\sin(x)\cos(x)^2\,dx$ | [*sum*] |
| $= -\cos(x) + \int -\sin(x)\cos(x)^2\,dx$ | [sin] |
| $= -\cos(x) - \left(\int \sin(x)\cos(x)^2\,dx\right)$ | [*constantmultiple*] |
| $= -\cos(x) - \left(\int -u^2\,du\right)$ | [*change*, $u = \cos(x)$, $u$] |
| $= -\cos(x) + \int u^2\,du$ | [*constantmultiple*] |
| $= -\cos(x) + \frac{u^3}{3}$ | [*power*] |
| $= -\cos(x) + \frac{\cos(x)^3}{3}$ | [*revert*] |

"стр.: 51, Высшая математика, 4. Киреева ЮГ, Петров ВВ" **(4)**

> *ShowSolution*(GetProblem(4, internal)); $mess_1$;

$$\int \cos(x)^2\sin(x)^3\,dx$$

| | |
|---|---|
| $= \int -\cos(x)^2(\cos(x)^2-1)\sin(x)\,dx$ | [*rewrite*, $\sin(x)^3 = \sin(x)-\sin(x)\cos(x)^2$] |
| $= \int (-\sin(x)\cos(x)^4+\sin(x)\cos(x)^2)\,dx$ | [*rewrite*, $-\cos(x)^2(\cos(x)^2-1)\sin(x) = -\sin(x)\cos(x)^4+\sin(x)\cos(x)^2$] |
| $= \int -\sin(x)\cos(x)^4\,dx + \int \sin(x)\cos(x)^2\,dx$ | [*sum*] |
| $= -\left(\int \sin(x)\cos(x)^4\,dx\right) + \int \sin(x)\cos(x)^2\,dx$ | [*constantmultiple*] |
| $= -\left(\int -u^4\,du\right) + \int \sin(x)\cos(x)^2\,dx$ | [*change*, $u = \cos(x)$, $u$] |
| $= \int u^4\,du + \int \sin(x)\cos(x)^2\,dx$ | [*constantmultiple*] |
| $= \frac{u^5}{5} + \int \sin(x)\cos(x)^2\,dx$ | [*power*] |
| $= \frac{\cos(x)^5}{5} + \int \sin(x)\cos(x)^2\,dx$ | [*revert*] |
| $= \frac{\cos(x)^5}{5} + \int -u^2\,du$ | [*change*, $u = \cos(x)$, $u$] |
| $= \frac{\cos(x)^5}{5} - \left(\int u^2\,du\right)$ | [*constantmultiple*] |
| $= \frac{\cos(x)^5}{5} - \frac{u^3}{3}$ | [*power*] |
| $= \frac{\cos(x)^5}{5} - \frac{\cos(x)^3}{3}$ | [*revert*] |

"стр.: 51, Высшая математика, 4. Киреева ЮГ, Петров ВВ" **(5)**

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