Examples

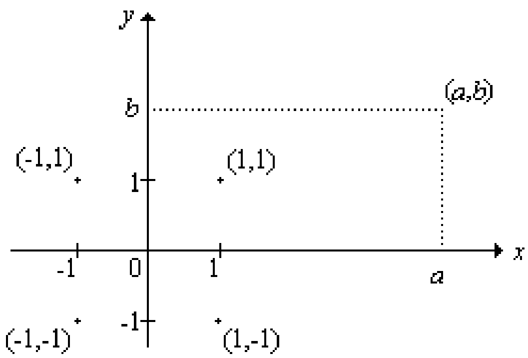
Уравнение Лапласа, часто встречается в физике

$$G_0 := \frac{\partial^2}{\partial x^2} U(x, y, z) + \frac{\partial^2}{\partial y^2} U(x, y, z) + \frac{\partial^2}{\partial z^2} U(x, y, z) \qquad (1.1)$$

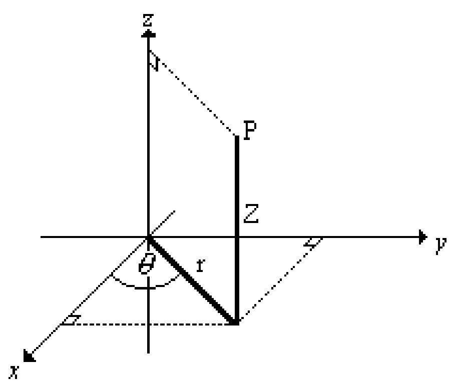
Тема: Якобиан преобразования и замена координат в уравнении Лапласа.

Примеры ортогональных координат

Декартовы координаты или прямоугольные координаты



Цилиндрические координаты



Сферические координаты

$$x = r\sin(\phi)\cos(\theta)$$

$$y = r\sin(\phi)\sin(\theta)$$

$$z = r\cos(\phi)$$

Примеры решения задач

Переход к цилиндрическим координатам

$$trcyl := \{x = r\cos(\theta), y = r\sin(\theta), U(x, y, z) = u(r, \theta, z)\} \quad (1.2)$$

Берем частные производные

$$dxch := \left\{ \frac{\partial}{\partial x} U(x, y, z) = \frac{\cos(\theta)\left(\frac{\partial}{\partial r} u(r, \theta, z)\right) r - \sin(\theta)\left(\frac{\partial}{\partial \theta} u(r, \theta, z)\right)}{r}, \frac{\partial}{\partial y} U(x, y, z) = \frac{\sin(\theta)\left(\frac{\partial}{\partial r} u(r, \theta, z)\right) r + \cos(\theta)\left(\frac{\partial}{\partial \theta} u(r, \theta, z)\right)}{r}, \frac{\partial}{\partial z} U(x, y, z) = \frac{\partial}{\partial z} u(r, \theta, z) \right\} \quad (1.3)$$

$$> vars := \left[\frac{\partial}{\partial r} u(r, \theta, z), \frac{\partial}{\partial \theta} u(r, \theta, z), \frac{\partial}{\partial z} u(r, \theta, z)\right] : oldvars := \left[\frac{\partial}{\partial x} U(x, y, z), \frac{\partial}{\partial y} U(x, y, z), \frac{\partial}{\partial z} U(x, y, z)\right] :$$

Линейная система уравнений для поиска Якобиана преобразования в матричной форме.

$$> A, b := GenerateMatrix\left(map\left(simplify, solve(dxch, vars)_1, 'symbolic'\right), oldvars\right)$$

$$A, b := \begin{bmatrix} -\cos(\theta) & -\sin(\theta) & 0 \\ r\sin(\theta) & -r\cos(\theta) & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -\left(\frac{\partial}{\partial r} u(r, \theta, z)\right) \\ -\left(\frac{\partial}{\partial \theta} u(r, \theta, z)\right) \\ -\left(\frac{\partial}{\partial z} u(r, \theta, z)\right) \end{bmatrix} \quad (1.4)$$

Находим Якобиан преобразования

$$> simplify(Determinant(A), 'symbolic')$$

$$-r \quad (1.5)$$

Переписываем все производные в новых цилиндрических координатах

$$\frac{\partial}{\partial x} U(x,y,z) = \frac{\cos(\theta)\left(\frac{\partial}{\partial r} u(r,\theta,z)\right) r - \sin(\theta)\left(\frac{\partial}{\partial \theta} u(r,\theta,z)\right)}{r} \quad (1.6)$$

$$\frac{\partial}{\partial y} U(x,y,z) = \frac{\sin(\theta)\left(\frac{\partial}{\partial r} u(r,\theta,z)\right) r + \cos(\theta)\left(\frac{\partial}{\partial \theta} u(r,\theta,z)\right)}{r} \quad (1.7)$$

$$\frac{\partial}{\partial z} U(x,y,z) = \frac{\partial}{\partial z} u(r,\theta,z) \quad (1.8)$$

$$\frac{\partial^2}{\partial x^2} U(x,y,z) = \frac{1}{r^2}\Bigg(\left(\frac{\partial^2}{\partial r^2} u(r,\theta,z)\right)\cos(\theta)^2 r^2 - 2\left(\frac{\partial^2}{\partial \theta\,\partial r} u(r,\theta,z)\right)\cos(\theta)\sin(\theta)\, r - \left(\frac{\partial}{\partial r} u(r,\theta,z)\right)\cos(\theta)^2 r - \left(\frac{\partial^2}{\partial \theta^2} u(r,\theta,z)\right)\cos(\theta)^2 + 2\left(\frac{\partial}{\partial \theta} u(r,\theta,z)\right)\cos(\theta)\sin(\theta) + \left(\frac{\partial}{\partial r} u(r,\theta,z)\right) r + \frac{\partial^2}{\partial \theta^2} u(r,\theta,z)\Bigg) \quad (1.9)$$

$$\frac{\partial^2}{\partial y^2} U(x,y,z) = \frac{1}{r^2}\Bigg(-\left(\frac{\partial^2}{\partial r^2} u(r,\theta,z)\right)\cos(\theta)^2 r^2 + \left(\frac{\partial}{\partial r} u(r,\theta,z)\right)\cos(\theta)^2 r + 2\left(\frac{\partial^2}{\partial \theta\,\partial r} u(r,\theta,z)\right)\cos(\theta)\sin(\theta)\, r + \left(\frac{\partial^2}{\partial \theta^2} u(r,\theta,z)\right)\cos(\theta)^2 - 2\left(\frac{\partial}{\partial \theta} u(r,\theta,z)\right)\cos(\theta)\sin(\theta) + \left(\frac{\partial^2}{\partial r^2} u(r,\theta,z)\right) r^2\Bigg) \quad (1.10)$$

$$\frac{\partial^2}{\partial z^2} U(x,y,z) = \frac{\partial^2}{\partial z^2} u(r,\theta,z) \quad (1.11)$$

Подставляем их значения в уравнение Лапласа в декартовых координатах и получаем запись уравнения Лапласа в цилиндрических координатах:

$$\frac{\partial^2}{\partial r^2} u(r,\theta,z) + \frac{\partial^2}{\partial z^2} u(r,\theta,z) + \frac{\frac{\partial}{\partial r} u(r,\theta,z)}{r} + \frac{\frac{\partial^2}{\partial \theta^2} u(r,\theta,z)}{r^2} \quad (1.12)$$

Переход к сферическим координатам

$$trsph := \{x = r\sin(\phi)\cos(\theta),\ y = r\sin(\phi)\sin(\theta),\ z = r\cos(\phi),\ U(x,y,z) = u(r,\theta,\phi)\} \quad (1.13)$$

$$> vars := \left[\frac{\partial}{\partial r} u(r,\theta,\phi), \frac{\partial}{\partial \theta} u(r,\theta,\phi), \frac{\partial}{\partial \phi} u(r,\theta,\phi)\right] : oldvars := \left[\frac{\partial}{\partial x} U(x,y,z), \frac{\partial}{\partial y} U(x,y,z), \frac{\partial}{\partial z} U(x,y,z)\right] :$$

Линейная система уравнений для поиска Якобиана преобразования:

$$A, b := \begin{bmatrix} -\sin(\phi)\cos(\theta) & -\sin(\phi)\sin(\theta) & -\cos(\phi) \\ r\sin(\phi)\sin(\theta) & -r\sin(\phi)\cos(\theta) & 0 \\ -r\cos(\phi)\cos(\theta) & -r\cos(\phi)\sin(\theta) & -\frac{r\left(\cos(\phi)^2 - 1\right)}{\sin(\phi)} \end{bmatrix}, \begin{bmatrix} -\left(\frac{\partial}{\partial r} u(r,\theta,\phi)\right) \\ -\left(\frac{\partial}{\partial \theta} u(r,\theta,\phi)\right) \\ -\left(\frac{\partial}{\partial \phi} u(r,\theta,\phi)\right) \end{bmatrix} \quad (1.14)$$

Якобиан преобразования

$$\sin(\phi)\, r^2 \quad (1.15)$$

Переписываем производные в новых сферических координатах

$$\frac{\partial}{\partial x} U(x,y,z) = \frac{1}{r\sin(\phi)}\left(\left(-r\cos(\phi)^2 + r\right)\cos(\theta)\left(\frac{\partial}{\partial r} u(r,\theta,\phi)\right) + \cos(\theta)\cos(\phi)\left(\frac{\partial}{\partial \phi} u(r,\theta,\phi)\right)\sin(\phi) - \sin(\theta)\left(\frac{\partial}{\partial \theta} u(r,\theta,\phi)\right)\right) \tag{1.16}$$

$$\frac{\partial}{\partial y} U(x,y,z) = \frac{1}{r\sin(\phi)}\left(\left(-r\cos(\phi)^2 + r\right)\sin(\theta)\left(\frac{\partial}{\partial r} u(r,\theta,\phi)\right) + \sin(\theta)\cos(\phi)\left(\frac{\partial}{\partial \phi} u(r,\theta,\phi)\right)\sin(\phi) + \cos(\theta)\left(\frac{\partial}{\partial \theta} u(r,\theta,\phi)\right)\right) \tag{1.17}$$

$$\frac{\partial}{\partial z} U(x,y,z) = \frac{\cos(\phi)\left(\frac{\partial}{\partial r} u(r,\theta,\phi)\right) r - \sin(\phi)\left(\frac{\partial}{\partial \phi} u(r,\theta,\phi)\right)}{r} \tag{1.18}$$

$$\frac{\partial^2}{\partial x^2} U(x,y,z) = \frac{1}{r^2\sin(\phi)^2}\left(r^2\cos(\theta)^2\left(\cos(\phi)-1\right)^2\left(\cos(\phi)+1\right)^2\left(\frac{\partial^2}{\partial r^2} u(r,\theta,\phi)\right) + \left(1 - \cos(\theta)^2\right)\left(\frac{\partial^2}{\partial \theta^2} u(r,\theta,\phi)\right) + \left(-\cos(\phi)^4 + \cos(\phi)^2\right)\cos(\theta)^2\left(\frac{\partial^2}{\partial \phi^2} u(r,\theta,\phi)\right) + 2 r\cos(\theta)\sin(\theta)\left(\cos(\phi)-1\right)\left(\cos(\phi)+1\right)\left(\frac{\partial^2}{\partial \theta\,\partial r} u(r,\theta,\phi)\right) - 2 r\cos(\phi)\sin(\phi)\cos(\theta)^2\left(\cos(\phi)-1\right)\left(\cos(\phi)+1\right)\left(\frac{\partial^2}{\partial r\,\partial \phi} u(r,\theta,\phi)\right) - 2\cos(\phi)\sin(\phi)\left(\frac{\partial^2}{\partial \theta\,\partial \phi} u(r,\theta,\phi)\right)\cos(\theta)\sin(\theta) - \left(\cos(\phi)-1\right)\left(1 + \left(\cos(\phi)^2 - 1\right)\cos(\theta)^2\right) r\left(\cos(\phi)+1\right)\left(\frac{\partial}{\partial r} u(r,\theta,\phi)\right) + 2\left(\frac{1}{2} + \left(\cos(\phi)^2 - \frac{3}{2}\right)\cos(\theta)^2\right)\cos(\phi)\sin(\phi)\left(\frac{\partial}{\partial \phi} u(r,\theta,\phi)\right) + 2\left(\frac{\partial}{\partial \theta} u(r,\theta,\phi)\right)\cos(\theta)\sin(\theta)\right) \tag{1.19}$$

$$\frac{\partial^2}{\partial y^2} U(x,y,z) = \frac{1}{r^2\sin(\phi)^2}\left(-r^2\left(\cos(\theta)-1\right)\left(\cos(\theta)+1\right)\left(\cos(\phi)-1\right)^2\left(\cos(\phi)+1\right)^2\left(\frac{\partial^2}{\partial r^2} u(r,\theta,\phi)\right) + \left(\left(\cos(\theta)^2-1\right)\cos(\phi)^4 + \left(1-\cos(\theta)^2\right)\cos(\phi)^2\right)\left(\frac{\partial^2}{\partial \phi^2} u(r,\theta,\phi)\right) + 2 r\cos(\phi)\sin(\phi)\left(\cos(\theta)-1\right)\left(\cos(\theta)+1\right)\left(\cos(\phi)-1\right)\left(\cos(\phi)+1\right)\left(\frac{\partial^2}{\partial r\,\partial \phi} u(r,\theta,\phi)\right) - 2 r\cos(\theta)\sin(\theta)\left(\cos(\phi)-1\right)\left(\cos(\phi)+1\right)\left(\frac{\partial^2}{\partial \theta\,\partial r} u(r,\theta,\phi)\right) + \left(\frac{\partial^2}{\partial \theta^2} u(r,\theta,\phi)\right)\cos(\theta)^2 + 2\cos(\phi)\sin(\phi)\left(\frac{\partial^2}{\partial \theta\,\partial \phi} u(r,\theta,\phi)\right)\cos(\theta)\sin(\theta) + \left(\cos(\phi)-1\right)\left(\left(\cos(\theta)^2 - 1\right)\cos(\phi)^2 - \cos(\theta)^2\right) r\left(\cos(\phi)+1\right)\left(\frac{\partial}{\partial r} u(r,\theta,\phi)\right) - 2\left(\left(\cos(\theta)^2 - 1\right)\cos(\phi)^2 - \frac{3}{2}\cos(\theta)^2 + 1\right)\cos(\phi)\sin(\phi)\left(\frac{\partial}{\partial \phi} u(r,\theta,\phi)\right) - 2\left(\frac{\partial}{\partial \theta} u(r,\theta,\phi)\right)\cos(\theta)\sin(\theta)\right) \tag{1.20}$$

$$\frac{\partial^2}{\partial z^2} U(x,y,z) = \frac{1}{r^2}\left(\cos(\phi)^2\left(\frac{\partial^2}{\partial r^2} u(r,\theta,\phi)\right) r^2 - 2\cos(\phi)\sin(\phi)\left(\frac{\partial^2}{\partial r\,\partial \phi} u(r,\theta,\phi)\right) r - \cos(\phi)^2\left(\frac{\partial}{\partial r} u(r,\theta,\phi)\right) r + 2\cos(\phi)\sin(\phi)\left(\frac{\partial}{\partial \phi} u(r,\theta,\phi)\right) - \cos(\phi)^2\left(\frac{\partial^2}{\partial \phi^2} u(r,\theta,\phi)\right)\right. \tag{1.21}$$

$$+\left(\frac{\partial}{\partial r}u(r,\theta,\phi)\right)r+\frac{\partial^2}{\partial\phi^2}u(r,\theta,\phi)\Big)$$

Подставляем их значения в уравнение Лапласа в декартовых координатах и получаем запись уравнения Лапласа в сферических координатах

$$\frac{1}{\sin(\phi)^2 r^2}\Bigg(-\cos(\phi)^2\left(\frac{\partial^2}{\partial r^2}u(r,\theta,\phi)\right)r^2-2\cos(\phi)^2\left(\frac{\partial}{\partial r}u(r,\theta,\phi)\right)r+\left(\frac{\partial^2}{\partial r^2}u(r,\theta,\phi)\right)r^2 + \cos(\phi)\sin(\phi)\left(\frac{\partial}{\partial\phi}u(r,\theta,\phi)\right)-\cos(\phi)^2\left(\frac{\partial^2}{\partial\phi^2}u(r,\theta,\phi)\right)+2\left(\frac{\partial}{\partial r}u(r,\theta,\phi)\right)r + \frac{\partial^2}{\partial\theta^2}u(r,\theta,\phi)+\frac{\partial^2}{\partial\phi^2}u(r,\theta,\phi)\Bigg) \tag{1.22}$$

Еще пример: переход к "экзотическим" биполярным цилиндрическим координатам: (Spiegel)

$$tr_0 := \left\{x=\frac{a\sinh(v)}{\cosh(v)-\cos(u)},\ y=\frac{a\sin(u)}{\cosh(v)-\cos(u)},\ z=w,\ U(x,y,z)=\mathcal{U}(u,v,w)\right\} \tag{1.23}$$

$$dxch := \Bigg\{\frac{\partial}{\partial x}U(x,y,z) = -\frac{\sinh(v)\sin(u)\left(\frac{\partial}{\partial u}\mathcal{U}(u,v,w)\right)-\left(\frac{\partial}{\partial v}\mathcal{U}(u,v,w)\right)+\left(\frac{\partial}{\partial v}\mathcal{U}(u,v,w)\right)\cosh(v)\cos(u)}{a},$$
$$\frac{\partial}{\partial y}U(x,y,z) = \frac{\left(\frac{\partial}{\partial u}\mathcal{U}(u,v,w)\right)\cosh(v)\cos(u)-\sinh(v)\sin(u)\left(\frac{\partial}{\partial v}\mathcal{U}(u,v,w)\right)-\left(\frac{\partial}{\partial u}\mathcal{U}(u,v,w)\right)}{a},$$
$$\frac{\partial}{\partial z}U(x,y,z)=\frac{\partial}{\partial w}\mathcal{U}(u,v,w)\Bigg\} \tag{1.24}$$

$$vars := \left[\frac{\partial}{\partial u}\mathcal{U}(u,v,w),\ \frac{\partial}{\partial v}\mathcal{U}(u,v,w),\ \frac{\partial}{\partial w}\mathcal{U}(u,v,w)\right] \tag{1.25}$$

$$oldvars := \left[\frac{\partial}{\partial x}U(x,y,z),\ \frac{\partial}{\partial y}U(x,y,z),\ \frac{\partial}{\partial z}U(x,y,z)\right] \tag{1.26}$$

$$A, b := \begin{bmatrix} \frac{\sinh(v)\sin(u)\,a}{\cosh(v)^2-2\cosh(v)\cos(u)+\cos(u)^2} & \frac{a\left(-\cosh(v)\cos(u)+1\right)}{\cosh(v)^2-2\cosh(v)\cos(u)+\cos(u)^2} & 0 \\ \frac{a\left(\cosh(v)\cos(u)-1\right)}{\cosh(v)^2-2\cosh(v)\cos(u)+\cos(u)^2} & \frac{\sinh(v)\sin(u)\,a}{\cosh(v)^2-2\cosh(v)\cos(u)+\cos(u)^2} & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -\left(\frac{\partial}{\partial u}\mathcal{U}(u,v,w)\right) \\ -\left(\frac{\partial}{\partial v}\mathcal{U}(u,v,w)\right) \\ -\left(\frac{\partial}{\partial w}\mathcal{U}(u,v,w)\right) \end{bmatrix} \tag{1.27}$$

Якобиан преобразования

$$-\frac{a^2}{\cosh(v)^2 - 2\cosh(v)\cos(u) + \cos(u)^2} \tag{1.28}$$

$$\frac{\partial}{\partial x} U(x, y, z) = \frac{-\sinh(v)\sin(u)\left(\frac{\partial}{\partial u}\mathcal{U}(u, v, w)\right) - \left(\frac{\partial}{\partial v}\mathcal{U}(u, v, w)\right)\cosh(v)\cos(u) + \frac{\partial}{\partial v}\mathcal{U}(u, v, w)}{a} \tag{1.29}$$

$$\frac{\partial}{\partial y} U(x, y, z) = \frac{(\cosh(v)\cos(u) - 1)\left(\frac{\partial}{\partial u}\mathcal{U}(u, v, w)\right) - \sinh(v)\sin(u)\left(\frac{\partial}{\partial v}\mathcal{U}(u, v, w)\right)}{a} \tag{1.30}$$

$$\frac{\partial}{\partial z} U(x, y, z) = \frac{\partial}{\partial w}\mathcal{U}(u, v, w) \tag{1.31}$$

$$\frac{\partial^2}{\partial x^2} U(x, y, z) = \frac{1}{a^2}\Bigg(\left(\left(1 - \cos(u)^2\right)\cosh(v)^2 + \cos(u)^2 - 1\right)\left(\frac{\partial^2}{\partial u^2}\mathcal{U}(u, v, w)\right) + (\cosh(v)\cos(u) - 1)^2\left(\frac{\partial^2}{\partial v^2}\mathcal{U}(u, v, w)\right) + 2\sin(u)\sinh(v)(\cosh(v)\cos(u) - 1)\left(\frac{\partial^2}{\partial v\,\partial u}\mathcal{U}(u, v, w)\right) + 2\left(\cos(u)\cosh(v)^2 - \frac{1}{2}\cos(u) - \frac{1}{2}\cosh(v)\right)\sin(u)\left(\frac{\partial}{\partial u}\mathcal{U}(u, v, w)\right) + 2\left(\frac{\partial}{\partial v}\mathcal{U}(u, v, w)\right)\left(\cosh(v)\cos(u)^2 - \frac{1}{2}\cos(u) - \frac{1}{2}\cosh(v)\right)\sinh(v)\Bigg) \tag{1.32}$$

$$\frac{\partial^2}{\partial y^2} U(x, y, z) = \frac{1}{a^2}\Bigg(\left(\left(1 - \cos(u)^2\right)\cosh(v)^2 + \cos(u)^2 - 1\right)\left(\frac{\partial^2}{\partial v^2}\mathcal{U}(u, v, w)\right) + (\cosh(v)\cos(u) - 1)^2\left(\frac{\partial^2}{\partial u^2}\mathcal{U}(u, v, w)\right) - 2\sin(u)\sinh(v)(\cosh(v)\cos(u) - 1)\left(\frac{\partial^2}{\partial v\,\partial u}\mathcal{U}(u, v, w)\right) - 2\left(\cos(u)\cosh(v)^2 - \frac{1}{2}\cos(u) - \frac{1}{2}\cosh(v)\right)\sin(u)\left(\frac{\partial}{\partial u}\mathcal{U}(u, v, w)\right) - 2\left(\frac{\partial}{\partial v}\mathcal{U}(u, v, w)\right)\left(\cosh(v)\cos(u)^2 - \frac{1}{2}\cos(u) - \frac{1}{2}\cosh(v)\right)\sinh(v)\Bigg) \tag{1.33}$$

$$\frac{\partial^2}{\partial z^2} U(x, y, z) = \frac{\partial^2}{\partial w^2}\mathcal{U}(u, v, w) \tag{1.34}$$

Подставляем их значения в уравнение Лапласа в декартовых координатах и получаем запись уравнения Лапласа в биполярных цилиндрических координатах

$$\frac{\cos(u)^2\left(\frac{\partial^2}{\partial v^2}\mathcal{U}(u, v, w)\right)}{a^2} + \frac{\cos(u)^2\left(\frac{\partial^2}{\partial u^2}\mathcal{U}(u, v, w)\right)}{a^2} - \frac{2\cosh(v)\cos(u)\left(\frac{\partial^2}{\partial v^2}\mathcal{U}(u, v, w)\right)}{a^2} - \frac{2\cosh(v)\cos(u)\left(\frac{\partial^2}{\partial u^2}\mathcal{U}(u, v, w)\right)}{a^2} + \frac{\cosh(v)^2\left(\frac{\partial^2}{\partial v^2}\mathcal{U}(u, v, w)\right)}{a^2} \tag{1.35}$$

$$+\frac{\cosh(v)^2\left(\frac{\partial^2}{\partial u^2}\mathcal{U}(u,v,w)\right)}{a^2}+\frac{\partial^2}{\partial w^2}\mathcal{U}(u,v,w)$$

> $NumTask := 69$

$$NumTask := 69 \tag{1}$$

Индивидуальные задания с ответами

> $vars := \left[\frac{\partial}{\partial u}Z(u,v,w), \frac{\partial}{\partial v}Z(u,v,w), \frac{\partial}{\partial w}Z(u,v,w)\right]:$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 1

Новые координаты:

$$\left\{x=\frac{2}{5}u+\frac{3}{5}v-\frac{2}{5}w, y=-\frac{4}{5}u-\frac{4}{5}v-\frac{1}{5}w, z=\frac{4}{5}u+\frac{1}{5}v+\frac{4}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{6775}{162}\frac{\partial^2}{\partial u^2}U(u,v,w)-\frac{6425}{81}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)-\frac{4550}{81}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{3125}{81}\frac{\partial^2}{\partial v^2}U(u,v,w)+\frac{4300}{81}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{1625}{81}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{18}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 2

Новые координаты:

$$\left\{x=-\frac{1}{5}u-\frac{1}{5}v-\frac{1}{5}w, y=\frac{4}{5}u-\frac{3}{5}v+\frac{4}{5}w, z=\frac{2}{5}v+\frac{4}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{11325}{784}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{975}{49}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)-\frac{5125}{392}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{425}{49}\frac{\partial^2}{\partial v^2}U(u,v,w)-\frac{425}{49}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{2925}{784}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{28}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 3

Новые координаты:

$$\left\{x=-\frac{3}{5}u+\frac{2}{5}v-\frac{4}{5}w, y=-\frac{1}{5}u-\frac{1}{5}v+\frac{1}{5}w, z=-\frac{4}{5}u+\frac{3}{5}v+\frac{3}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{2275}{484}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{5525}{484}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)+\frac{175}{484}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{16875}{1936}\frac{\partial^2}{\partial v^2}U(u,v,w)+\frac{425}{968}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{1875}{1936}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{44}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 4

Новые координаты:

$$\left\{x=\frac{1}{5}\,v-\frac{2}{5}\,w,\,y=-\frac{2}{5}\,u+\frac{1}{5}\,v,\,z=\frac{2}{5}\,u+\frac{4}{5}\,v-\frac{4}{5}\,w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{25}{4}\,\frac{\partial^2}{\partial u^2}\,U(u,v,w)+\frac{25}{3}\,\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)+\frac{25}{2}\,\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)+\frac{50}{3}\,\frac{\partial^2}{\partial v^2}\,U(u,v,w)+\frac{100}{3}\,\frac{\partial^2}{\partial w\,\partial v}\,U(u,v,w)+\frac{75}{4}\,\frac{\partial^2}{\partial w^2}\,U(u,v,w)$$

Якобиан:

$$-\frac{12}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 5

Новые координаты:

$$\left\{x=\frac{4}{5}\,u-\frac{2}{5}\,v+\frac{3}{5}\,w,\,y=-\frac{3}{5}\,v,\,z=-\frac{4}{5}\,u+\frac{4}{5}\,v+\frac{3}{5}\,w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{75}{32}\,\frac{\partial^2}{\partial u^2}\,U(u,v,w)-\frac{25}{18}\,\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)+\frac{275}{162}\,\frac{\partial^2}{\partial w^2}\,U(u,v,w)+\frac{25}{6}\,\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)+\frac{25}{9}\,\frac{\partial^2}{\partial v^2}\,U(u,v,w)-\frac{50}{27}\,\frac{\partial^2}{\partial w\,\partial v}\,U(u,v,w)$$

Якобиан:

$$\frac{72}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 6

Новые координаты:

$$\left\{x=\frac{3}{5}\,u-\frac{3}{5}\,v-\frac{1}{5}\,w,\,y=-\frac{2}{5}\,u-\frac{1}{5}\,v+\frac{1}{5}\,w,\,z=\frac{4}{5}\,u-\frac{1}{5}\,v-\frac{3}{5}\,w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{50}{3}\,\frac{\partial^2}{\partial u^2}\,U(u,v,w)+\frac{25}{2}\,\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)+\frac{275}{6}\,\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)+\frac{125}{24}\,\frac{\partial^2}{\partial v^2}\,U(u,v,w)+\frac{175}{12}\,\frac{\partial^2}{\partial w\,\partial v}\,U(u,v,w)+\frac{275}{8}\,\frac{\partial^2}{\partial w^2}\,U(u,v,w)$$

Якобиан:

$$-\frac{12}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 7

Новые координаты:

$$\left\{x=-\frac{3}{5}\,u-\frac{2}{5}\,v-\frac{2}{5}\,w,\,y=-\frac{1}{5}\,u-\frac{1}{5}\,v+\frac{1}{5}\,w,\,z=\frac{2}{5}\,u-\frac{3}{5}\,w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{1525}{121}\,\frac{\partial^2}{\partial u^2}\,U(u,v,w)-\frac{5050}{121}\,\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)+\frac{1300}{121}\,\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)+\frac{4875}{121}\,\frac{\partial^2}{\partial v^2}\,U(u,v,w)-\frac{2450}{121}\,\frac{\partial^2}{\partial w\,\partial v}\,U(u,v,w)+\frac{525}{121}\,\frac{\partial^2}{\partial w^2}\,U(u,v,w)$$

Якобиан:

$$\frac{11}{125}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 8
Новые координаты:

$$\left\{x = -\frac{3}{5}\, v - \frac{2}{5}\, w, y = \frac{1}{5}\, u + \frac{4}{5}\, w, z = -\frac{4}{5}\, u - \frac{1}{5}\, v - \frac{2}{5}\, w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{25}{11}\, \frac{\partial^2}{\partial u^2}\, U(u, v, w) - \frac{25}{11}\, \frac{\partial^2}{\partial w\, \partial u}\, U(u, v, w) + \frac{75}{22}\, \frac{\partial^2}{\partial v^2}\, U(u, v, w) - \frac{25}{11}\, \frac{\partial^2}{\partial w\, \partial v}\, U(u, v, w) + \frac{175}{88}\, \frac{\partial^2}{\partial w^2}\, U(u, v, w)$$

Якобиан:

$$-\frac{44}{125}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 9
Новые координаты:

$$\left\{x = -\frac{1}{5}\, u + \frac{2}{5}\, v + \frac{2}{5}\, w, y = -\frac{4}{5}\, u + \frac{2}{5}\, v - \frac{4}{5}\, w, z = -\frac{3}{5}\, u - \frac{4}{5}\, v - \frac{1}{5}\, w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{350}{169}\, \frac{\partial^2}{\partial u^2}\, U(u, v, w) - \frac{175}{507}\, \frac{\partial^2}{\partial v\, \partial u}\, U(u, v, w) - \frac{1700}{507}\, \frac{\partial^2}{\partial w\, \partial u}\, U(u, v, w) + \frac{6425}{6084}\, \frac{\partial^2}{\partial v^2}\, U(u, v, w) + \frac{425}{1521}\, \frac{\partial^2}{\partial w\, \partial v}\, U(u, v, w) + \frac{3875}{1521}\, \frac{\partial^2}{\partial w^2}\, U(u, v, w)$$

Якобиан:

$$-\frac{78}{125}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 10
Новые координаты:

$$\left\{x = \frac{2}{5}\, u + \frac{4}{5}\, v + \frac{3}{5}\, w, y = \frac{4}{5}\, u + \frac{1}{5}\, v + \frac{2}{5}\, w, z = \frac{3}{5}\, u - \frac{2}{5}\, v - \frac{2}{5}\, w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{275}{243}\, \frac{\partial^2}{\partial u^2}\, U(u, v, w) + \frac{700}{243}\, \frac{\partial^2}{\partial v\, \partial u}\, U(u, v, w) - \frac{1000}{243}\, \frac{\partial^2}{\partial w\, \partial u}\, U(u, v, w) + \frac{3575}{243}\, \frac{\partial^2}{\partial v^2}\, U(u, v, w) - \frac{7900}{243}\, \frac{\partial^2}{\partial w\, \partial v}\, U(u, v, w) + \frac{4775}{243}\, \frac{\partial^2}{\partial w^2}\, U(u, v, w)$$

Якобиан:

$$-\frac{27}{125}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 11
Новые координаты:

$$\left\{x = -\frac{3}{5}\, u - \frac{3}{5}\, v - \frac{4}{5}\, w, y = -\frac{2}{5}\, u - \frac{1}{5}\, w, z = -\frac{2}{5}\, u - \frac{4}{5}\, v + \frac{3}{5}\, w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{8125}{968}\, \frac{\partial^2}{\partial u^2}\, U(u, v, w) - \frac{5525}{484}\, \frac{\partial^2}{\partial v\, \partial u}\, U(u, v, w) - \frac{625}{121}\, \frac{\partial^2}{\partial w\, \partial u}\, U(u, v, w) + \frac{4725}{968}\, \frac{\partial^2}{\partial v^2}\, U(u, v, w) + \frac{425}{121}\, \frac{\partial^2}{\partial w\, \partial v}\, U(u, v, w) + \frac{425}{242}\, \frac{\partial^2}{\partial w^2}\, U(u, v, w)$$

Якобиан:

$$\frac{44}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 12

Новые координаты:

$$\left\{x = -\frac{2}{5}u + \frac{3}{5}v + \frac{3}{5}w, y = \frac{2}{5}u + \frac{1}{5}w, z = \frac{2}{5}u - \frac{4}{5}v\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{25}{4}\frac{\partial^2}{\partial u^2}U(u,v,w) + \frac{100}{13}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w) - \frac{25}{13}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w) + \frac{50}{13}\frac{\partial^2}{\partial v^2}U(u,v,w) - \frac{50}{13}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w) + \frac{50}{13}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{26}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 13

Новые координаты:

$$\left\{x = \frac{1}{5}u + \frac{1}{5}v + \frac{1}{5}w, y = -\frac{1}{5}u + \frac{4}{5}w, z = \frac{4}{5}u + \frac{4}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{425}{16}\frac{\partial^2}{\partial v^2}U(u,v,w) + 2\left(\frac{\partial^2}{\partial u^2}U(u,v,w)\right) - \frac{3}{2}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w) + \frac{17}{16}\frac{\partial^2}{\partial w^2}U(u,v,w) - \frac{5}{2}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w) - \frac{5}{8}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)$$

Якобиан:

$$-\frac{4}{25}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 14

Новые координаты:

$$\left\{x = -\frac{3}{5}w, y = -\frac{1}{5}u + \frac{4}{5}v - \frac{2}{5}w, z = \frac{2}{5}u - \frac{4}{5}v - \frac{1}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$75\left(\frac{\partial^2}{\partial u^2}U(u,v,w)\right) + \frac{175}{3}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w) + \frac{50}{3}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w) + \frac{875}{72}\frac{\partial^2}{\partial v^2}U(u,v,w) + \frac{125}{18}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w) + \frac{25}{9}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{12}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 15

Новые координаты:

$$\left\{x = \frac{4}{5}u + \frac{4}{5}w, y = \frac{2}{5}u + \frac{4}{5}v + \frac{1}{5}w, z = -\frac{4}{5}u - \frac{3}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{625}{16}\frac{\partial^2}{\partial u^2}U(u,v,w) - \frac{275}{16}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w) - \frac{175}{2}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w) + \frac{225}{64}\frac{\partial^2}{\partial v^2}U(u,v,w)$$

$$+\frac{75}{4}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+50\left(\frac{\partial^2}{\partial w^2}U(u,v,w)\right)$$

Якобиан:

$$-\frac{16}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 16

Новые координаты:

$$\left\{x=-\frac{4}{5}u-\frac{2}{5}v-\frac{2}{5}w,\ y=\frac{4}{5}u+\frac{4}{5}v-\frac{3}{5}w,\ z=-\frac{3}{5}u-\frac{2}{5}v\right\}$$

Уравнение Лапласа в декартовых координатах:

$$1550\left(\frac{\partial^2}{\partial u^2}U(u,v,w)\right)-4475\left(\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)\right)-1800\left(\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)\right)+\frac{12925}{4}\frac{\partial^2}{\partial v^2}U(u,v,w)+2600\left(\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)\right)+525\left(\frac{\partial^2}{\partial w^2}U(u,v,w)\right)$$

Якобиан:

$$\frac{2}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 17

Новые координаты:

$$\left\{x=-\frac{2}{5}u+\frac{3}{5}v-\frac{3}{5}w,\ y=-\frac{2}{5}u-\frac{4}{5}v+\frac{2}{5}w,\ z=\frac{1}{5}u+\frac{1}{5}v+\frac{3}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{2350}{529}\frac{\partial^2}{\partial u^2}U(u,v,w)-\frac{1700}{529}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)-\frac{2150}{529}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{4325}{2116}\frac{\partial^2}{\partial v^2}U(u,v,w)+\frac{3525}{1058}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{5625}{2116}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{46}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 18

Новые координаты:

$$\left\{x=-\frac{1}{5}u-\frac{2}{5}w,\ y=\frac{3}{5}u-\frac{4}{5}v+\frac{1}{5}w,\ z=\frac{3}{5}u+\frac{4}{5}v+\frac{2}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{425}{81}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{275}{162}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)-\frac{1100}{81}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{625}{648}\frac{\partial^2}{\partial v^2}U(u,v,w)-\frac{475}{162}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{950}{81}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{36}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 19

Новые координаты:

$$\left\{x=\frac{3}{5}u-\frac{3}{5}v-\frac{2}{5}w,\ y=\frac{2}{5}u-\frac{1}{5}v+\frac{2}{5}w,\ z=\frac{2}{5}u-\frac{3}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{154}{25}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{62}{5}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)+\frac{72}{25}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+9\left(\frac{\partial^2}{\partial v^2}U(u,v,w)\right)+\frac{8}{5}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{49}{25}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{1}{5}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 20

Новые координаты:

$$\left\{x=\frac{1}{5}v-\frac{1}{5}w, y=-\frac{4}{5}u+\frac{2}{5}v, z=-\frac{4}{5}u-\frac{1}{5}v+\frac{2}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{525}{16}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{275}{2}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)+\frac{375}{2}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+150\left(\frac{\partial^2}{\partial v^2}U(u,v,w)\right)+400\left(\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)\right)+275\left(\frac{\partial^2}{\partial w^2}U(u,v,w)\right)$$

Якобиан:

$$\frac{4}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 21

Новые координаты:

$$\left\{x=\frac{3}{5}u-\frac{3}{5}v-\frac{3}{5}w, y=\frac{2}{5}u+\frac{4}{5}v+\frac{4}{5}w, z=\frac{1}{5}u-\frac{4}{5}v-\frac{1}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{625}{324}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{100}{81}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)-\frac{175}{162}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{250}{81}\frac{\partial^2}{\partial v^2}U(u,v,w)-\frac{550}{81}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{1525}{324}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{54}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 22

Новые координаты:

$$\left\{x=\frac{3}{5}u+\frac{3}{5}v, y=\frac{1}{5}u-\frac{4}{5}v-\frac{3}{5}w, z=\frac{2}{5}u+\frac{1}{5}v+\frac{2}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{3550}{1521}\frac{\partial^2}{\partial u^2}U(u,v,w)-\frac{3850}{1521}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)+\frac{400}{169}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{4525}{1521}\frac{\partial^2}{\partial v^2}U(u,v,w)-\frac{1050}{169}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{875}{169}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{39}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 23

Новые координаты:

$$\left\{x = -\frac{1}{5}\ v + \frac{1}{5}\ w, y = -\frac{1}{5}\ u - \frac{1}{5}\ v - \frac{2}{5}\ w, z = \frac{2}{5}\ u + \frac{1}{5}\ v + \frac{1}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{175}{8}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) - \frac{125}{4}\ \frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w) - \frac{75}{4}\ \frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w) + \frac{175}{8}\ \frac{\partial^2}{\partial v^2}\ U(u, v, w) + \frac{25}{4}\ \frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w) + \frac{75}{8}\ \frac{\partial^2}{\partial w^2}\ U(u, v, w)$$

Якобиан:

$$-\frac{4}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 24

Новые координаты:

$$\left\{x = \frac{2}{5}\ u - \frac{3}{5}\ v - \frac{4}{5}\ w, y = \frac{3}{5}\ v + \frac{2}{5}\ w, z = -\frac{1}{5}\ u - \frac{2}{5}\ v - \frac{3}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{775}{128}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) - \frac{25}{32}\ \frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w) + \frac{175}{64}\ \frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w) + \frac{375}{32}\ \frac{\partial^2}{\partial v^2}\ U(u, v, w) - \frac{625}{32}\ \frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w) + \frac{1175}{128}\ \frac{\partial^2}{\partial w^2}\ U(u, v, w)$$

Якобиан:

$$\frac{16}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 25

Новые координаты:

$$\left\{x = \frac{3}{5}\ u + \frac{3}{5}\ v - \frac{2}{5}\ w, y = -\frac{1}{5}\ u + \frac{4}{5}\ v - \frac{3}{5}\ w, z = -\frac{3}{5}\ u - \frac{3}{5}\ v - \frac{2}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{217}{72}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) - \frac{137}{36}\ \frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w) - \frac{15}{4}\ \frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w) + \frac{157}{72}\ \frac{\partial^2}{\partial v^2}\ U(u, v, w) + \frac{15}{4}\ \frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w) + \frac{25}{8}\ \frac{\partial^2}{\partial w^2}\ U(u, v, w)$$

Якобиан:

$$\frac{12}{25}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 26

Новые координаты:

$$\left\{x = -\frac{3}{5}\ u + \frac{4}{5}\ v + \frac{4}{5}\ w, y = -\frac{4}{5}\ u - \frac{2}{5}\ v - \frac{3}{5}\ w, z = \frac{4}{5}\ u - \frac{4}{5}\ v - \frac{3}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{425}{81}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) + \frac{3400}{81}\ \frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w) - \frac{3100}{81}\ \frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w) + \frac{15625}{162}\ \frac{\partial^2}{\partial v^2}\ U(u, v, w) - \frac{14425}{81}\ \frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w) + \frac{6725}{81}\ \frac{\partial^2}{\partial w^2}\ U(u, v, w)$$

Якобиан:

$$-\frac{18}{125}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 27
Новые координаты:

$$\left\{x=\frac{3}{5}u+\frac{2}{5}v-\frac{4}{5}w, y=-\frac{4}{5}u+\frac{4}{5}v-\frac{3}{5}w, z=-\frac{3}{5}u-\frac{2}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{45}{49}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{85}{49}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)+\frac{40}{49}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{475}{98}\frac{\partial^2}{\partial v^2}U(u,v,w)+\frac{310}{49}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{145}{49}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{14}{25}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 28
Новые координаты:

$$\left\{x=-\frac{4}{5}u-\frac{1}{5}v-\frac{4}{5}w, y=\frac{1}{5}u-\frac{1}{5}v+\frac{4}{5}w, z=-\frac{1}{5}u+\frac{3}{5}v\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{50}{11}\frac{\partial^2}{\partial u^2}U(u,v,w)-\frac{125}{22}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{25}{11}\frac{\partial^2}{\partial v^2}U(u,v,w)+\frac{225}{88}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{44}{125}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 29
Новые координаты:

$$\left\{x=-\frac{1}{5}u+\frac{1}{5}v-\frac{1}{5}w, y=\frac{3}{5}u+\frac{4}{5}v+\frac{4}{5}w, z=\frac{2}{5}u-\frac{2}{5}v\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{825}{49}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{950}{49}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)-\frac{300}{7}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{1725}{196}\frac{\partial^2}{\partial v^2}U(u,v,w)-\frac{425}{14}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{125}{4}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{14}{125}$$

ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 30
Новые координаты:

$$\left\{x=-\frac{2}{5}u+\frac{4}{5}v-\frac{3}{5}w, y=-\frac{2}{5}u-\frac{2}{5}v-\frac{1}{5}w, z=\frac{4}{5}u+\frac{1}{5}v+\frac{2}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{2875}{72}\frac{\partial^2}{\partial u^2}U(u,v,w)-\frac{350}{3}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{175}{2}\frac{\partial^2}{\partial w^2}U(u,v,w)-\frac{400}{9}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)+\frac{125}{9}\frac{\partial^2}{\partial v^2}U(u,v,w)+\frac{200}{3}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)$$

Якобиан:

$$\frac{12}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 31
Новые координаты:

$$\left\{x = \frac{1}{5}\ u - \frac{4}{5}\ v + \frac{2}{5}\ w, y = -\frac{1}{5}\ v, z = -\frac{3}{5}\ v - \frac{3}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{8425}{9}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) + 300\left(\frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w)\right) - \frac{2800}{9}\ \frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w) + 25\left(\frac{\partial^2}{\partial v^2}\ U(u, v, w)\right) - 50\left(\frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w)\right) + \frac{250}{9}\ \frac{\partial^2}{\partial w^2}\ U(u, v, w)$$

Якобиан:

$$-\frac{3}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 32
Новые координаты:

$$\left\{x = \frac{2}{5}\ u - \frac{3}{5}\ w, y = -\frac{1}{5}\ u - \frac{4}{5}\ v - \frac{2}{5}\ w, z = \frac{1}{5}\ u + \frac{4}{5}\ v - \frac{3}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{43}{4}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) - \frac{37}{8}\ \frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w) + \frac{83}{64}\ \frac{\partial^2}{\partial v^2}\ U(u, v, w) + 6\left(\frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w)\right) - \left(\frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w)\right) + 2\left(\frac{\partial^2}{\partial w^2}\ U(u, v, w)\right)$$

Якобиан:

$$-\frac{8}{25}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 33
Новые координаты:

$$\left\{x = \frac{4}{5}\ u - \frac{4}{5}\ w, y = \frac{2}{5}\ v + \frac{4}{5}\ w, z = \frac{2}{5}\ u - \frac{4}{5}\ v + \frac{3}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{5025}{2704}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) + \frac{575}{338}\ \frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w) + \frac{725}{676}\ \frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w) + \frac{1125}{676}\ \frac{\partial^2}{\partial v^2}\ U(u, v, w) + \frac{125}{169}\ \frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w) + \frac{525}{676}\ \frac{\partial^2}{\partial w^2}\ U(u, v, w)$$

Якобиан:

$$-\frac{104}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 34
Новые координаты:

$$\left\{x = -\frac{2}{5}\ u + \frac{3}{5}\ v - \frac{3}{5}\ w, y = -\frac{2}{5}\ u + \frac{1}{5}\ v - \frac{1}{5}\ w, z = -\frac{1}{5}\ u - \frac{2}{5}\ v + \frac{3}{5}\ w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{125}{8}\ \frac{\partial^2}{\partial u^2}\ U(u, v, w) + \frac{425}{4}\ \frac{\partial^2}{\partial v\,\partial u}\ U(u, v, w) + \frac{325}{4}\ \frac{\partial^2}{\partial w\,\partial u}\ U(u, v, w) + \frac{1825}{8}\ \frac{\partial^2}{\partial v^2}\ U(u, v, w) + \frac{1425}{4}\ \frac{\partial^2}{\partial w\,\partial v}\ U(u, v, w) + \frac{1125}{8}\ \frac{\partial^2}{\partial w^2}\ U(u, v, w)$$

Якобиан:

$$-\frac{4}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 35

Новые координаты:

$$\left\{x=-\frac{4}{5}\,u-\frac{1}{5}\,v-\frac{2}{5}\,w,\, y=\frac{2}{5}\,u+\frac{4}{5}\,v-\frac{4}{5}\,w,\, z=\frac{3}{5}\,v+\frac{4}{5}\,w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{5825}{3364}\,\frac{\partial^2}{\partial u^2}\,U(u,v,w)-\frac{1350}{841}\,\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)-\frac{75}{841}\,\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)+\frac{1125}{841}\,\frac{\partial^2}{\partial v^2}\,U(u,v,w)$$
$$+\frac{125}{841}\,\frac{\partial^2}{\partial w\,\partial v}\,U(u,v,w)+\frac{1175}{1682}\,\frac{\partial^2}{\partial w^2}\,U(u,v,w)$$

Якобиан:

$$\frac{116}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 36

Новые координаты:

$$\left\{x=\frac{1}{5}\,u-\frac{2}{5}\,v-\frac{4}{5}\,w,\, y=-\frac{1}{5}\,u-\frac{2}{5}\,v-\frac{2}{5}\,w,\, z=\frac{4}{5}\,u-\frac{4}{5}\,v-\frac{4}{5}\,w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{225}{16}\,\frac{\partial^2}{\partial v^2}\,U(u,v,w)-\frac{75}{4}\,\frac{\partial^2}{\partial w\,\partial v}\,U(u,v,w)+\frac{275}{36}\,\frac{\partial^2}{\partial w^2}\,U(u,v,w)+\frac{125}{36}\,\frac{\partial^2}{\partial u^2}\,U(u,v,w)$$
$$+\frac{25}{4}\,\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)-\frac{25}{18}\,\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)$$

Якобиан:

$$\frac{24}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 37

Новые координаты:

$$\left\{x=\frac{1}{5}\,u-\frac{1}{5}\,v,\, y=-\frac{1}{5}\,u-\frac{2}{5}\,v+\frac{4}{5}\,w,\, z=\frac{2}{5}\,u-\frac{1}{5}\,v\right\}$$

Уравнение Лапласа в декартовых координатах:

$$50\left(\frac{\partial^2}{\partial u^2}\,U(u,v,w)\right)+150\left(\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)\right)+100\left(\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)\right)+125\left(\frac{\partial^2}{\partial v^2}\,U(u,v,w)\right)$$
$$+\frac{325}{2}\,\frac{\partial^2}{\partial w\,\partial v}\,U(u,v,w)+\frac{875}{16}\,\frac{\partial^2}{\partial w^2}\,U(u,v,w)$$

Якобиан:

$$\frac{4}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 38

Новые координаты:

$$\left\{x=\frac{4}{5}\,u-\frac{2}{5}\,v-\frac{3}{5}\,w,\, y=-\frac{3}{5}\,u+\frac{2}{5}\,v+\frac{4}{5}\,w,\, z=\frac{1}{5}\,u+\frac{2}{5}\,v-\frac{2}{5}\,w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{31}{2}\,\frac{\partial^2}{\partial u^2}\,U(u,v,w)+11\left(\frac{\partial^2}{\partial v\,\partial u}\,U(u,v,w)\right)+24\left(\frac{\partial^2}{\partial w\,\partial u}\,U(u,v,w)\right)+\frac{39}{8}\,\frac{\partial^2}{\partial v^2}\,U(u,v,w)$$

$$+\frac{13}{2}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{21}{2}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{4}{25}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 39

Новые координаты:

$$\left\{x=\frac{1}{5}u-\frac{3}{5}v+\frac{2}{5}w,y=-\frac{2}{5}u-\frac{1}{5}v+\frac{3}{5}w,z=-\frac{4}{5}u+\frac{1}{5}v-\frac{4}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{3750}{2401}\frac{\partial^2}{\partial u^2}U(u,v,w)-\frac{550}{2401}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)-\frac{3350}{2401}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{11625}{2401}\frac{\partial^2}{\partial v^2}U(u,v,w)$$
$$+\frac{10650}{2401}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{5150}{2401}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$-\frac{49}{125}$$

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ОТВЕТ К РЕШЕНИЮ ЗАДАЧИ # 40

Новые координаты:

$$\left\{x=-\frac{3}{5}u+\frac{4}{5}v-\frac{4}{5}w,y=\frac{4}{5}u-\frac{1}{5}v+\frac{1}{5}w,z=-\frac{3}{5}u+\frac{3}{5}v+\frac{1}{5}w\right\}$$

Уравнение Лапласа в декартовых координатах:

$$\frac{425}{169}\frac{\partial^2}{\partial u^2}U(u,v,w)+\frac{1675}{338}\frac{\partial^2}{\partial v\,\partial u}U(u,v,w)+\frac{75}{338}\frac{\partial^2}{\partial w\,\partial u}U(u,v,w)+\frac{11075}{2704}\frac{\partial^2}{\partial v^2}U(u,v,w)$$
$$+\frac{3775}{1352}\frac{\partial^2}{\partial w\,\partial v}U(u,v,w)+\frac{6475}{2704}\frac{\partial^2}{\partial w^2}U(u,v,w)$$

Якобиан:

$$\frac{52}{125}$$

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