Thermo-mechanical model of geysers

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***Abstract-* A mathematical model describing dynamical patterns which probably may give start to a geyser eruption is analyzed. This appears as a simple oscillatory cavity resonator governed sensitively by small changes in the temperature-dependent viscosity of subsoil. Some typical patterns of the thermo-mechanical instability are traced parametrically in detail.**

Keywords- Thermo-mechanical instability; Cavity resonator; Resonance; Nonlinear oscillotions

I. INTRODUCTION

The nature exhibits, on the one hand, that active geyser eruptions include at lot of appropriate circumstances while on the other hand, the dynamics may be quite silent even under almost the same environment conditions. These dynamical processes traditionally require some theoretical explanations. The present paper is one of many attempts along this way [1–7]. From a physical viewpoint, it is obvious that the activity of geyser always causes a decrease in the viscosity of the subsoil fluid. Then the amplitude of thermo-mechanical oscillations should increase. Equations governing the motion of a cavity resonator, provided by a sufficiently large coefficient of energy dissipation, are proposed to play a role of the mathematical model in initial stages of the geyser dynamics. It is assumed that the coefficient of energy dissipation depends upon the ambient temperature and simple temperature dependence between the energy dissipation and the temperature is selected as a linear function with a small slope, which enters both into the heat balance equation and that describing mechanical vibrations. These equations are investigated using the small-parameter method under the assumption that the external excitation from side of subsoil is so small that it cannot cause significant large-amplitude non-linear oscillations. The study of steady-state oscillatory modes reveals dynamic processes treated as dangerous from a viewpoint of the beginning the geyser eruption. These are explicable in terms of thermo-mechanical instability of the nonlinear system in the vicinity of resonance.

A physical picture of dynamic processes in almost active geysers seams to be very simple. A drop in the viscosity leads to some increase of the amplitude, which contributes to some additional heat portion. This heat causes some decrease in the viscosity, so that the heat injection should be reduced. It is clear that such a process should be saturated and would approach some stationary state. However, the system under consideration, being nonlinear, has hysteretic steady-state regimes of motion, which can lead to dangerous oscillation regimes even being far from the resonant frequency. Such a situation is modelled along the specific examples following the parametric analysis performed to identify the most interesting oscillatory patterns.

Problems of the thermo-mechanical stability are of interest for physicists both on many traditional and novel areas. For example, the problems of ultrasonic techniques [8], phase transitions in austenite, the dynamics of materials with memory [9], electromechanical systems [10] cannot be traced without some adequate description between the thermal and mechanical effects. Questions of the thermo-mechanical stability in the light of geyser eruptions have been methodically studied both within variety of purely descriptive models and methodically, using exact thermo-dynamical approaches [1–7, 11–15]. Most theoretical models are restricted to a “Pyrex flask” that is heated by a burner. There are no objections to these scenarios. But, such models cannot explain, for instance, correlations in periodicity of the geyser activity. This paper tries to generalize the physical description by including the mechanical motion into the model, since the “Pyrex flask” may be treated as a damped cavity resonator. The dynamical response of this resonator depends highly sensitively upon extremely small variations in subsoil viscosity. The model is so simple that can represent just a possible additional fragment to generalise serious models describing geyser eruptions.

II. EQUATIONS OF MOTION

The influence of temperature effects upon the amplitude-frequency dependence describing steady-state oscillations is investigated. The equations of motion are based on the most basic general physical assumptions, briefly mentioned in the introduction:

$$\dot{x} = y;$$

$$m\dot{y} + 2\delta\left(1 - \alpha T\right)y + c(1 + \beta x)x = -\left(P + \mu p \sin\omega t\right);$$

$$CV\dot{T} = 2\delta\left(1 - \alpha T\right)y^2 - VG(T - \theta_0) + Q,$$

where the reduced coefficients of the model of a geyser activities are following: m is the mass of the cavity resonator, c is the coefficient of elasticity; α is the thermal coefficient of viscosity; β is the coefficient of elasticity, which characterizes the asymmetry of the deformation; V stands for the volume of the cavity; P denotes the static load of subsoil; p is the maximal value of the external harmonic force at the given frequency ω ; μ is some small dimensionless parameter modelling the rate of seismic activity. These equations, making allowances for the thermal balance, are also characterized by the following parameters: C is the heat capacity; G is the thermal conductivity; $x(t)$ denotes the mechanical displacement; $T(t)$ stands for the temperature; θ_0 is the ambient temperature; Q is the external heat power.

We define the static deformation under the static load P; $\Delta = -c + \sqrt{c^2 - 4\beta P}/2c\beta$ and the natural frequency of oscillation, $\varpi = \sqrt{\sqrt{(c^2 - 4\beta P)}/m}$, in the absence of energy dissipation. Then the following dimensionless variables are introduced:

$$\tau = \varpi t; \quad \lambda(\tau) = \frac{T(t) - T_0}{\mu T_0};$$
$$X(\tau) = \frac{-2x(t)c\beta - c + \sqrt{c^2 - 4\beta P}}{\mu lc\beta};$$
$$Y(\tau) = y(t)/\mu l\varpi .$$

Here $l = \sqrt[3]{V}$ is the characteristic length scale; $T_0 = \theta_0 + Q/G$ is the equilibrium temperature. The equations of motion in these dimensionless variables can be rewritten as:

$$\dot{X} = Y;$$

$$\dot{Y} + X + \frac{2\delta}{m\varpi}(1 - \alpha T_0)Y + \frac{p}{ml\varpi^2}\sin\left(\frac{\omega t}{\varpi}\right) = -\mu\frac{\left(c\beta l^2 X^2 - 2\alpha T_0 \delta l\varpi Y\lambda\right)}{ml\varpi^2}; \tag{1}$$

$$\dot{\lambda} + \frac{G}{C\varpi}\lambda = \mu\frac{2\delta l^2\varpi^2(1 - \alpha T_0)}{CV\varpi T_0}Y^2.$$

Upper dots denote a differentiation with respect to the dimensionless time τ.

The general solution to the linear subset (1), as $\mu \to 0$, can be obtained by any student of the second year of study. This solution is characterized three integration constants, say: A is an arbitrary complex constant ($\overline{A}$ corresponds to the complex conjugate amplitude of the cavity resonator) while B is a real arbitrary constant, respective for the temperature.

Finally, we introduce the resonant frequency at which the linear system reaches the amplitude peak at the same frequency of the external excitation; $\Omega = \left(m^2\varpi^2 - \delta^2(1 - \alpha T_0)^2\right)^{1/2}/m$.

III. Equations Evolution equations

To construct the first-order nonlinear approximation asymptotic solution as series in the small parameter μ, the paradigm of the method of arbitrary constant variations, known from courses of differential equations, is used. Formally, the same form of solutions, which satisfy the linear homogeneous subset, is explored:

$$X(\tau) = A(\tau)\exp\left(\frac{-\delta(1 - \alpha T_0) + im\Omega}{m\varpi}\right)\tau + \overline{A}(\tau)\exp\left(\frac{-\delta(1 - \alpha T_0) - im\Omega}{m\varpi}\right)\tau + u_0(\tau) + \mu u_1(\tau);$$

$$Y(\tau) = \dot{X}(\tau) \tag{2}$$

$$\lambda(\tau) = B(\tau)\exp\left(-\frac{G\tau}{C\varpi}\right) + w_0(\tau) + \mu w_1(\tau).$$

All the formerly "old" constants, $A = A(\tau)$, $\overline{A} = \overline{A}(\tau)$, $B = B(\tau)$, are now varying at the time τ; the functions $u_j(\tau)$, $v_j(\tau)$, $w_j(\tau)$ represent the so-called nonresonant corrections ($j = 0,1$). The order of approximations is determined by the index j. This one should be fully compatible with a standard expansion of the sought function as a series in μ. The nonresonant corrections are introduced to develop an asymptotic solution utilising an appropriate recursive method, due to the smallness of the parameter μ.

The following polar coordinates, $a(\tau)$ and $\varphi(\tau)$, describing the amplitude and phase, correspondingly, are introduced:

$$A(\tau) = a(\tau)\exp i\varphi(\tau)\exp\left(\frac{-\delta(1 - \alpha T_0)\tau}{m\varpi}\right);$$
$$\overline{A}(\tau) = a(\tau)\exp(-i\varphi(\tau))\exp\left(\frac{-\delta(1 - \alpha T_0)\tau}{m\varpi}\right). \tag{3}$$

This transform allows us to trace the so-called "fast" and "slow" motions near the resonance provided that the external excitation of the system (1) is small. The "fast" variable is characterized by the frequency of the external harmonic force ω, while the new phase $\psi(\tau) = \varphi(\tau) - (\omega - \Omega)\tau/\varpi$ plays as the "slow" variable. The difference $\omega - \Omega$ is associated with the phase-matching condition. This means that $\psi(\tau)$ should be a small value of order μ.

A. First-order nonlinear approximation equations

The average value of a given function $f(\tau)$ over the period $2\pi\varpi/\omega$ is determined by the following expression

$$\langle f\rangle = \frac{\omega}{2\pi\varpi}\int_0^{\frac{2\pi\varpi}{\omega}} f d\tau . \tag{4}$$

The averaging is performed over the "slow" variables when the resonance is removed from the system. After substituting (2) and (3) into eqs.(1), the averaging operator (4) leads to the following zero-order approximation evolution equations

$$\dot{a} + \frac{\delta(1-\alpha T_0)}{m\varpi} a - \frac{p\cos\psi}{4\varpi\, lm\Omega} = 0;$$

$$(5)\qquad \dot{\psi} + \frac{\omega - \Omega}{\varpi} + \frac{p\sin\psi(\tau)}{4a(\tau)\varpi\, lm\Omega} = 0;$$

$$\dot{B} = 0.$$

Stationary solutions to the set (5) are obtained by equating all the derivatives to zero:

$$a_0^2 = \frac{p^2}{16 l^2 m^2 \Omega^2\left(\left(\varpi^2 + \omega^2 - 2\omega\Omega\right)\right)};$$

$$(6)\qquad \psi_0 = -\arctan\left(\frac{m\omega - \Omega}{\delta(1-\alpha T_0)}\right);$$

$$B_0 = 0.$$

Here a_0, ψ_0, B_0 denote the steady-state values of the sought variables. Equations describing the zero-order corrections are following

$$\dot{u}_0 = v_0 + \frac{p\cos\left(\frac{\omega\tau}{\varpi}\right)}{l\varpi\sqrt{m^2\varpi^2/2 - \delta^2(1-\alpha T_0)^2}};$$

$$(7)\qquad \dot{v}_0 = -\frac{2\delta(1-\alpha T_0)}{m\varpi} v_0 - u_0 + \frac{p\delta(1-\alpha T_0)\cos\left(\frac{\omega\tau}{\varpi}\right)}{2\varpi^2 ml\sqrt{m^2\varpi^2/2 - \delta^2(1-\alpha T_0)^2}} - \frac{p\sin\left(\frac{\omega\tau}{\varpi}\right)}{2\varpi^2 ml};$$

$$w_0(\tau) = 0.$$

After finding any particular solution to eqs.(7), the zero-order approximation is completely built. It is obvious that the zero-order approximation stationary solution, in terms of the substitution (2), coincides exactly with the corresponding solution to the original linear subset (1).

To construct the nonlinear first-order approximation evolution equations, we can again use the same substitution (2), pointing out that the zero-order non-resonant correction is already are known as a particular solution to the inhomogeneous linear differential equations (7).

The evolution equations within the first-order non-linear approximation hold true:

$$\dot{a} - \frac{p\cos\psi}{4m\varpi\Omega l} + \frac{\delta(1-\alpha T_0)}{m\varpi} a\cos\psi = - \mu B\left(\gamma_{10} a + \gamma_{11}\sin\psi + \gamma_{12}\cos\psi\right);$$

$$(8)\qquad \dot{\psi} + \frac{\omega - \Omega}{\varpi} + \frac{p}{4m\varpi\Omega l}\frac{\sin\psi}{a} = - \mu B\left(\gamma_{20} + \gamma_{21}\frac{\sin\psi}{a} + \gamma_{22}\frac{\cos\psi}{a}\right);$$

$$\dot{B} + \frac{G}{C\varpi} B = - \mu\left(\gamma_{30} + a\left(\gamma_{31}\sin\psi + \gamma_{32}\cos\psi\right) + \gamma_{33} a^2\right).$$

The expressions for the coefficients to eqs.(8) are so long, but these are written exactly in [16].

The structure of the first-order approximation evolution equations (8) is very transparent. It is obvious that the intensity of the thermo-mechanical effect is determined by the small parameter μ . If this parameter is zero, then there is no any temperature effect on the mechanical motion. If we assume the thermal viscosity parameter α to be zero, then the coefficients γ_{10}, γ_{11}, γ_{12} and γ_{20}, γ_{21}, γ_{22} should be also zeros in the equations for the amplitude a and phase ψ . There is no any temperature effect on the mechanical motion again. Let us remove these limiting cases from consideration and the nontrivial nonlinear thermo-mechanical coupling becomes apparent.

B. Phase-amplitude frequency response with thermal effects

The equations determining the steady-state oscillatory modes follow directly from the evolution equations (8) if we put all the velocities equal to zero. As a result one obtains the set of three transcendental equations for the same number of unknowns variables $\hat{a}$, $\hat{\psi}$ and $\hat{B}$:

$$\frac{p\cos\hat{\psi}}{4m\varpi\Omega l} + \frac{\delta(1-\alpha T_0)}{m\varpi}\hat{a}\cos\hat{\psi} = - \mu\hat{B}\left(\gamma_{10}\hat{a} + \gamma_{11}\sin\hat{\psi} + \gamma_{12}\cos\hat{\psi}\right);$$

$$(9)\qquad \frac{\omega - \Omega}{\varpi} + \frac{p}{4m\varpi\Omega l}\frac{\sin\hat{\psi}}{\hat{a}} = - \mu\hat{B}\left(\gamma_{20} + \gamma_{21}\frac{\sin\hat{\psi}}{\hat{a}} + \gamma_{22}\frac{\cos\hat{\psi}}{\hat{a}}\right);$$

$$\frac{G}{C\varpi}\hat{B} = - \mu\left(\gamma_{30} + \hat{a}\left(\gamma_{31}\sin\hat{\psi} + \gamma_{32}\cos\hat{\psi}\right) + \gamma_{33}\hat{a}^2\right).$$

The unknown quantities, $\hat{a}$, $\hat{\psi}$ and $\hat{B}$, characterizing the amplitude, phase and the temperature, respectively, can be parameterized in different ways. Let these be the functions of the external frequency ω . Then one can build the so-called amplitude-phase frequency curves, taking into account temperature effects. For clarity, we may consider the specific values of individual parameters to the system (1) for some specific volume of active geyser subsoil. Let the values of these hypothetic parameters be: $V = 10^{-6} m^3$; $m = 1.0\,kg$; $c = 10^5\,Nm^{-1}$; $\delta = 400\,N\,sm^{-1}$; $P = 10^3\,N$; $p = 100\,N$; $\alpha = 10^{-3}\,K^{-1}$; $\beta = 1.0\,m^{-1}$; $G = 10^2\,WK^{-1}m^{-3}$; $T_0 = 380K$; $l = 0.01\,m$. Then there is the possibility to trace the behaviour of the amplitude and phase characteristics of the stationary processes depending upon the small parameter μ. The steady-state characteristics, when the parameter is small enough, e.g. $\mu = 10^{-2}$, are shown in Fig. 1. The frequency in all the pictures is normalized by the value $\varpi = 313\,Hz$, so that the maximal amplitude should be near unity. The amplitude, $\hat{a}(\nu)$, and temperature, $\hat{B}(\nu)$, are presented as functions of the dimensionless frequency $\nu = \omega / \varpi$.

It is obvious that the set of stationary states is composed of two distinct subsets, namely **H** and **L**, which are appropriate to call the high- and low-temperature branches, respectively. The amplitude- and phase-frequency branches, characterizing the low-temperature subset **L**, are almost indistinguishable from the related curves (6), characterizing the linear subsystem (Fig. 2). At the same time the high-temperature subset **H** appears entirely due to the nonlinearity. This subset consists of both stable and unstable fixed points separated by limits where the derivatives become infinite. Obviously, the stable stationary regimes **H** cannot be reachable from any initial conditions. For example, to excite any stable high-temperature stationary regime the liquid inside the geyser should be unnaturally pre-heated up to some predetermined temperature. Moreover, the frequency of the external harmonic signal should be within the specified band. At the same time the stationary regimes correspondent to the low-temperature subset **L** are achieved almost at any initial conditions.

Obviously, that the temperature related to the high-temperature branch **H** should be most sensitive to changes in the small parameter μ. The amplitude varies slowly than the temperature, but the resonant peak is shifted slightly into the high-frequency band. In turn, the high-temperature branch **H** changes very rapidly with the growth of the small parameter. Starting with a certain critical value of this parameter, the high-temperature characteristic **H** is united totally with the low-temperature branch **L**. This causes the *thermo-mechanical instability* of the system, which is expressed in a high jump in the oscillation amplitude and a significant increase of the temperature, both in the vicinity of the resonance frequency and even some higher. Figure 3 illustrates the stationary states near the critical point. The path (a, b, c, d, e, a) in this figure represents the hysteresis loop when the external frequency ω is scanned to and fro.

The system under consideration (8) is complex enough to evaluate analytically their stability properties. Nonetheless, numerical tests over the set (1) can confirm oscillatory patterns naturally observed in systems with a hysteresis.

Let the small parameter μ increases. How significant are the changes over the amplitude and temperature characteristics? The figure 2 exhibits a significant nonlinear dependence even within the low-temperature steady-state solutions upon a relatively small variations in the parameter μ. Point out that the high-temperature branch is not shown in this figure.

Moreover, we should not forget that Fig. 3 demonstrates the results provided by the first-order approximation nonlinear model (8), though, direct numerical calculations of the original equations of motion (1) in some characteristic points confirm that the thermo-mechanical instability actually takes place. It turns out that the solution to the first-order nonlinear approximation equations (8) practically coincides with those of the initial problem (1) at small amplitudes in the vicinity of the resonant frequency. Some discrepancies between the exact and approximate solutions naturally increase with the growth of the external periodic load. It means that the second-order nonlinear approximation equations play an actual role from the viewpoint of a more detailed description of the frequency-amplitude dependences. But this question, being a nontrivial one, is beyond the scope of present study.

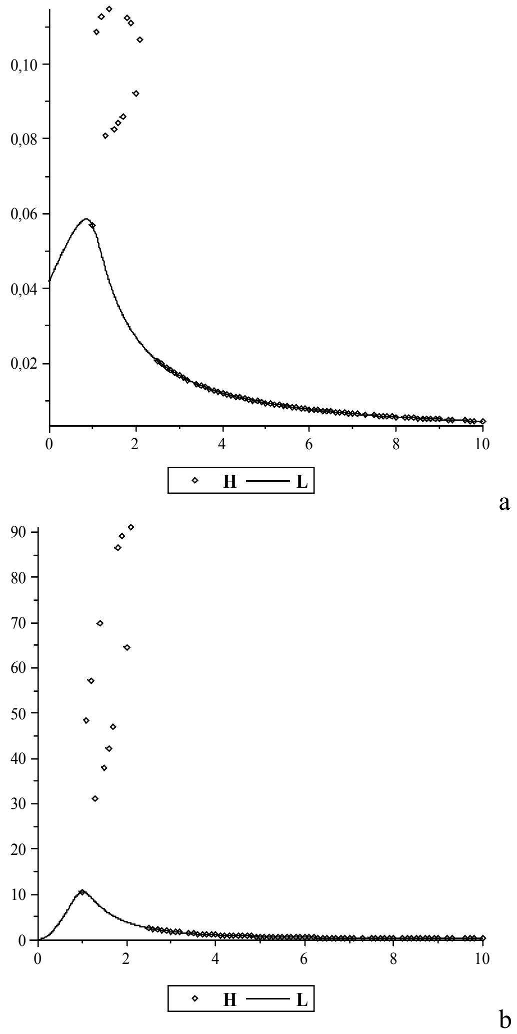


Fig. 1 Amplitude (a) and temperature (b) as functions of the dimensionless frequency ν.

IV. Parametric analysis of stationary solutions

To carry out a parametric analysis of stationary solutions to the nonlinear evolution equations of the first-order approximation, the left-hand side of eqs.(8) are indicated as $P(\hat{a},\hat{\psi},\hat{B})$, $Q(\hat{a},\hat{\psi},\hat{B})$ and $R(\hat{a},\hat{\psi},\hat{B})$, correspondingly. The unknown quantities, $\hat{a}$, $\hat{\psi}$ and $\hat{B}$, describing, as before, the amplitude, the phase and temperature, respectively, are now considered to be smooth functions of the small parameter μ. It is obvious that expression (9) represent explicit solutions to the eqs.(8) with the initial conditions defined by the known parameters, $\hat{a}(0)$, $\hat{\psi}(0)$ and $\hat{B}(0)$. The functions P, Q and R are differentiable almost everywhere in the space of the system parameters. Then the parametric analysis of stationary solutions is available with the help of the Lie series [17, 18]. These functions should be once differentiated by the variable μ in eqs.(9), then these obtained equations are resolved to the implicit set for the first derivatives. The result appears as the following three ordinary differential equations:

$$\begin{aligned} &\frac{d\hat{a}}{d\mu} = \zeta_{\hat{a}}\left(\hat{a},\hat{\psi},\hat{B}\right);\\ (10)\quad &\frac{d\hat{\psi}}{d\mu} = \zeta_{\hat{\psi}}\left(\hat{a},\hat{\psi},\hat{B}\right);\\ &\frac{d\hat{B}}{d\mu} = \zeta_{\hat{B}}\left(\hat{a},\hat{\psi},\hat{B}\right). \end{aligned}$$

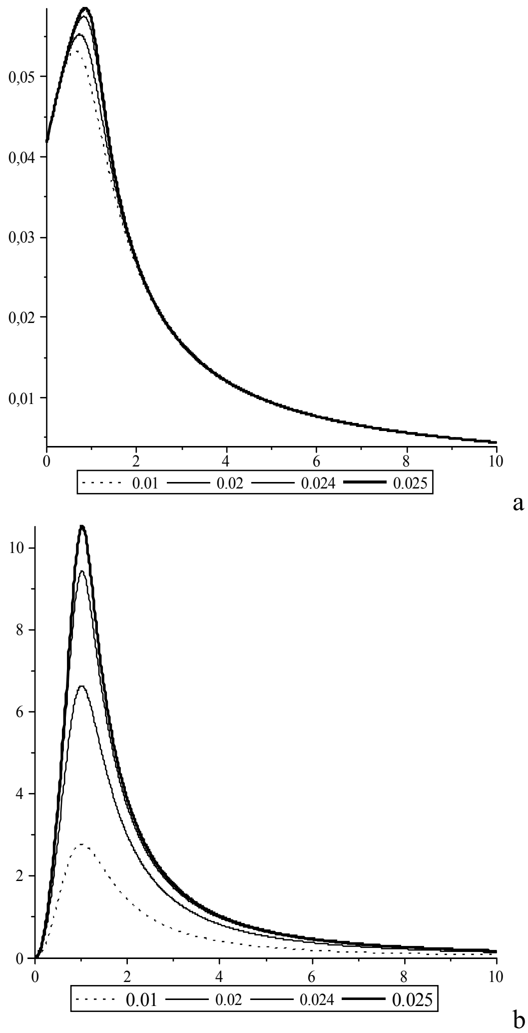


Fig. 2 Low-temperature branch L. Amplitude (a) and temperature (b). Numbers marked with different values of the small parameter.

Evidently, the initial conditions to these equations are completely determined by the right-hand sides of steady states (6), i.e. $\hat{a}(0) = a_0$, $\hat{\psi}(0) = \psi_0$ and $\hat{B}(0) = B_0$. The structure of these equations is not so easy, but it can be effectively studied in detail using parsing algorithms [16].

V. Dependence of steady-state solutions upon the small parameter μ

The numerical result to eqs.(10) is shown in Fig. 4, as an illustrative example. The values of the system parameters are the same as previously.

Peaks of the displacement and temperature are formed even away from the resonant frequency Ω, as we can see in Fig. 4. A typical resonant pattern takes place when the frequency of the external signal, ω, tends to the resonant frequency of the system Ω.

C. Steady-states versus the nonlinear elastic parameter β

The steady states obtained by scanning the parameter β, characterizing the asymmetry of the elastic characteristics are shown in Fig. 5. There is no any significant impact on the dynamics of the geyser dynamics even with a very significant change in this parameter.

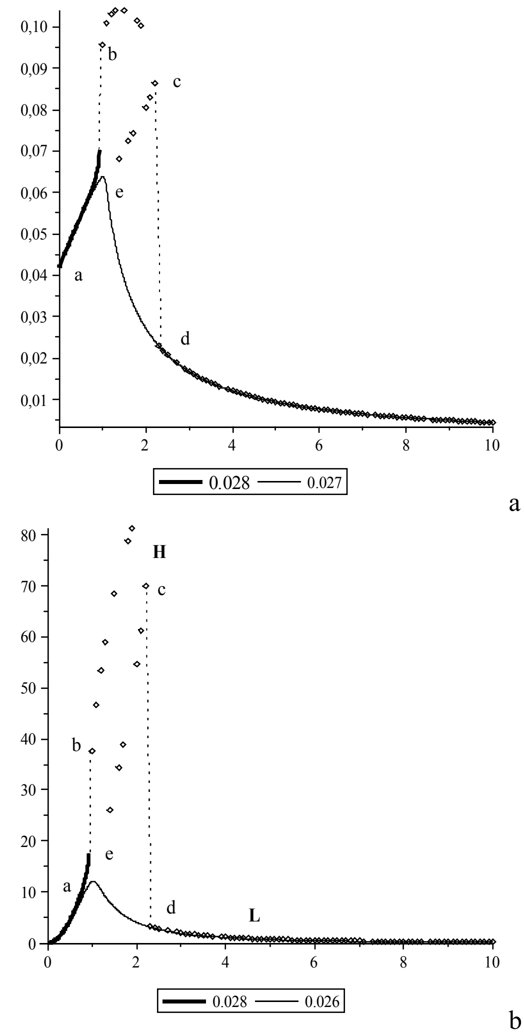


Fig. 3 Thermo-mechanical instability. Amplitude response (a) and temperature (b) versus the dimensionless frequency ν. Numbers marked with different values of the small parameter.

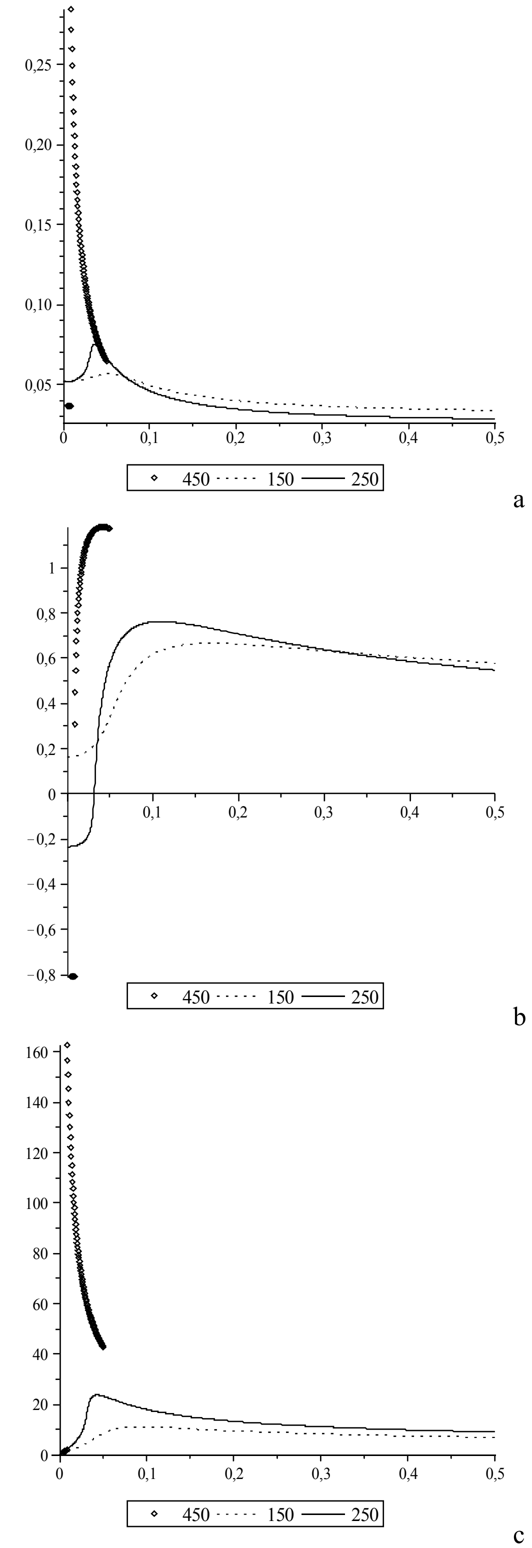


Fig. 4 Stationary solutions. Amplitude (a), phase (b) and temperature (c) versus the small parameter μ. Numbers marked with the calculated values of the external signal frequency in Hertz.

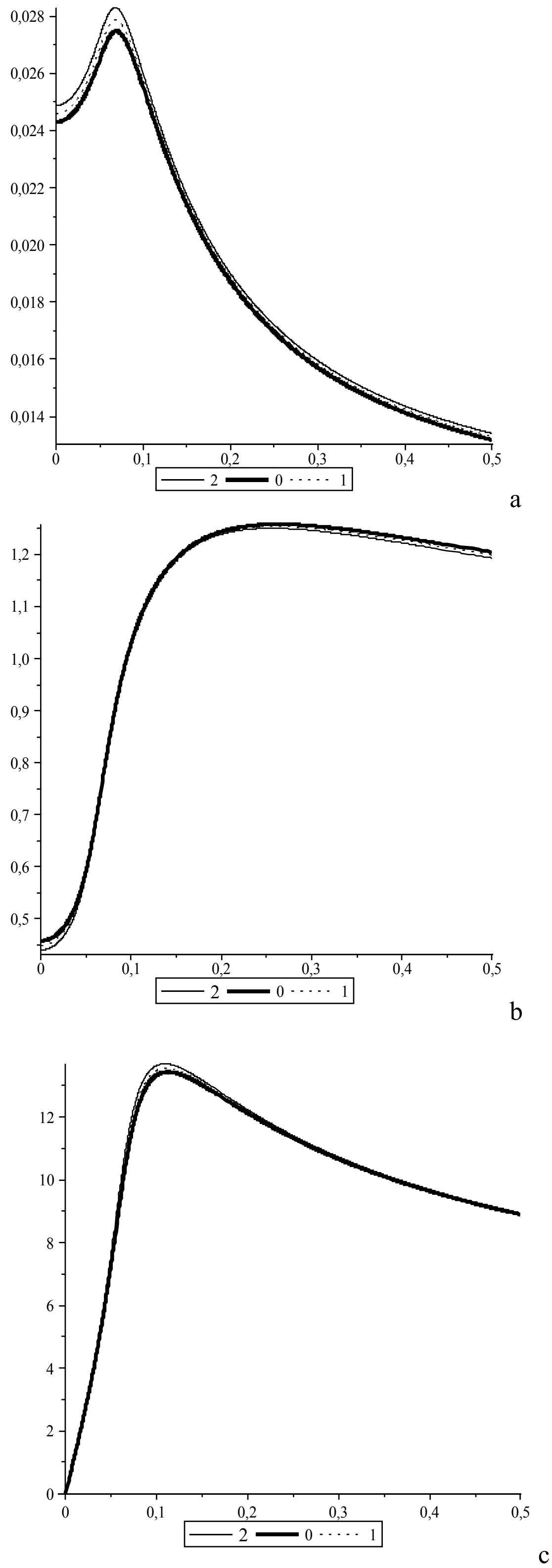
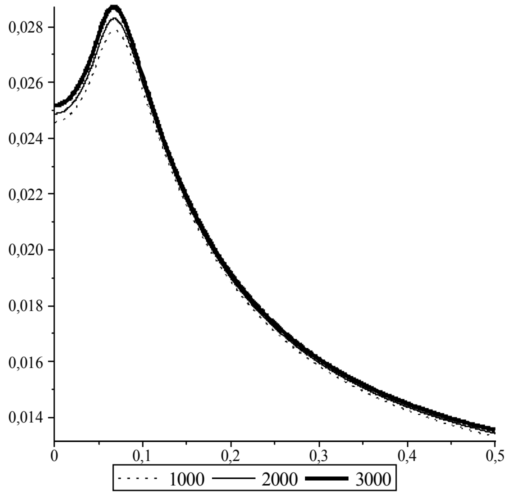


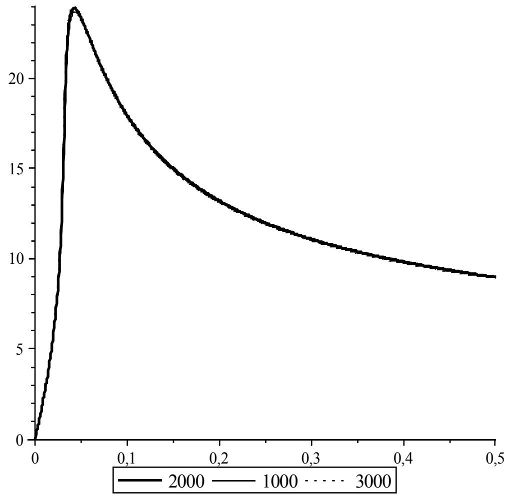
Fig. 5 Stationary states. Amplitude (a), phase (b) and temperature (c) versus the small parameter μ. Numbers are marked with the calculated values for the nonlinear elasticity (units $1/m$).

D. Steady-states versus the static load P

The steady states along the variable parameter of the static load P, characterizing a constant component of the subsoil pressure, are illustrated in Fig. 6. It is clearly that the static load influences significantly on the geyser dynamics.



a



c

Fig. 6 Stationary states. Amplitude (a) and temperature (b) versus the small parameter. Numbers are marked with the calculated values of the static load (units Newton).

VI. Conclusions

A mathematical model describing initial stages of the geyser thermo-mechanical instability has been traced in this paper. This instability is caused by the nonlinear resonant phenomena. The temperature of the fluid inside the geyser cavity, which is regarded as a damped mechanical resonator, increases under extremely small external harmonic excitation, so that, the viscosity decreases while the amplitude of mechanical vibrations increases, as well. This decrease in viscosity causes some restriction in the heat injection. This leads to the nonlinear steady states. In the vicinity of the resonant frequency, the system exhibits a strong amplitude-frequency dependence, which provides some hysteretic patterns of oscillations. Parametric analysis of the system reveals that the value of thermal viscosity is the most sensitive parameter from the viewpoint of the thermo-mechanical instability. It is demonstrated that this parameter approaches a critical value at extremely small variations in the system. In practice, this means that some even small impurities, such as particles of rocks or salt concentration, can critically influence upon the geyser dynamics. Possibly, this can help to explain some evident periodicity in eruptions observed in the nature.

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