SYNCHRONIZATION OF A PAIR OF DRIVERS ON AN ELASTIC FOUNDATION

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Abstract

In this paper we present the results of theoretical studies inspired by the problem to reduce vibrations and noise using hydraulic absorbers as a damper to dissipate the energy of vibrations in the railway electric equipment of the track ВЛ-8ОС. The results of initial experimental studies on reducing the energy consumption of induction rotors, and some theoretical calculations demonstrated in this paper show the ability to customize the damping properties of hydraulic absorbers saving a power and saving the equipment due to a self-locking mode of rotation of rotors.

Introduction

The phenomenon of the phase synchronization, being first physically described by Huygens, begins intensively studied mathematically only since the mid 20-th century, in parallel with significant advances in electronics [1-4]. Fundamental results on the synchronization in terms of the qualitative theory of differential equations and bifurcation theory [5] prove the resonance nature of this phenomenon. Now the application of this theory is widely used to solve pressing practical problems in a wide range of activities, from microelectronics to power supply [6-8]. Now the interests of researchers in advanced fields of synchronization theory concentrated, apparently due to the rapid development of new technologies, for studying complex systems with chaotic dynamics, discrete objects and systems with time delay variables. However, in the traditional areas of human activity such as, for instance, energy and transport, is also noticeable growth of interest in this phenomenon, in order to find effective ways to save energy and integrity of the power units. The progressive development of science is constantly improving and expanding our understanding of the phenomenon of synchronization, as a consistent coherent dynamic process. It occurs usually due to very small, almost imperceptible bonds between the individual elements of the system, which, nevertheless, a qualitative change in the dynamical behavior of the object.

The basic equation of the theory of phase synchronization of a pair of oscillators (rotator) reads

$$\frac{d\Psi}{dT} = \delta + Q\sin\Psi ,$$

where δ is a small frequency (or angular velocity) detuning, Q is the depth of the phase modulation, T is the time. This one being a very simple equation has the general solution in the following form

$$\Psi(T) = 2\arctan\left(\frac{1}{\delta}\left(\tan\left(\frac{T}{2}\sqrt{\delta^2 - Q^2} + \frac{C}{2}\sqrt{\delta^2 - Q^2}\right)\sqrt{\delta^2 - Q^2} - Q\right)\right),$$

where C is an arbitrary constant of integration, T - is the temporal scale. From this solution follows a simple stability criterion for the phase synchronization:

$$\delta^2 - Q^2 < 0.$$

It shows that the phase mismatch must be small, or, accordingly, the parameter of modulation must be sufficiently large, otherwise the synchronization may be destroyed.

A more detailed mathematical study of the classical problem of the stable synchronization in a two-rotor system based on an elastic foundation turns out that the reduced model is incomplete. Namely, it draws attention to itself by the fact that the model lacks any description of the element of the system by which proper and carried out the interaction of the rotors. It turned out that the structure of the refined model is

$$\frac{d\rho}{dT} = (S - D)\rho, \quad \frac{d\Psi}{dT} = \delta + Q\sin\Psi + R\rho^2,$$

where ρ describes a measure of the amplitude of oscillations of the elastic foundation. This additional equation appears as a result of the phase modulation of the angular frequency of rotors due to vibrations of the base. So that, the perturbed rotors, in turn, cause the resonant excitation of vibrations of the base, described by the second equation. In the study of the refined model explains that for the stable synchronization requires the addition condition, $\delta^2 - Q^2 < 0$. One more necessary condition is that the coefficient of the resonant excitation of vibrations of the base S should not exceed the rate of energy dissipation D, i.e. $S < D$. The last restriction significantly alters the stability region of the synchronization in the parameter space of the system that, in fact, will be demonstrated by specific computational examples.

The equations of motion

We consider the motion of two electric asynchronous engines mounted on an elastic platform. A mathematical model is presented by the following system of differential equations [6-8]

$$m\ddot{\eta} + p\eta - m_1 r_1\left(\ddot{\varphi}_1 \sin\varphi_1 + \dot{\varphi}_1^{\,2}\cos\varphi_1\right) - m_2 r_2\left(\ddot{\varphi}_2 \sin\varphi_2 + \dot{\varphi}_2^{\,2}\cos\varphi_2\right) = 0;$$

(1) $$I_1\ddot{\varphi}_1 + H_1(\varphi_1, \dot{\varphi}_1) - L_1(\varphi_1, \dot{\varphi}_1) - m_1 r_1 \ddot{\eta}\sin\varphi_1 = 0;$$

$$I_2\ddot{\varphi}_2 + H_2(\varphi_2, \dot{\varphi}_2) - L_2(\varphi_2, \dot{\varphi}_2) - m_2 r_2 \ddot{\eta}\sin\varphi_2 = 0,$$

where m is the mass of the platform, modeled as a rigid body with one degree of freedom, characterized by a linear horizontal displacement η, p is the coefficient of elasticity of the platform associated with a fixed base (ground), m_i are the small masses of eccentrics with the eccentricities r_i (the radiuses of inertia), I_i - are the moments of inertia of rotors in the absence of imbalance, $L_i(\varphi, \dot{\varphi})$ stands for the driving moments, $H_i(\varphi, \dot{\varphi})$ denotes the resistance moment of the rotor. It is installed a pair of the asynchronous engines (unbalanced rotors) on the platform, whose axis is perpendicular to the direction of oscillation. The angles of rotation of the rotor φ_i measured from the direction of the axis η counter-clockwise. Assume that the moment characteristics of each engine and torque resistance have a simple linearised form, i.e. $L_i(\varphi_i, \dot{\varphi}_i) = M_i - k_i\dot{\varphi}_i$, $H_i(\varphi_i, \dot{\varphi}_i) = k_{0i}\dot{\varphi}_i$. Here M_i are the constant parameters, respective for the starting points, k_{0i} and k_i stand for the drag coefficients of the devices. Respectively, the subscript “1” refers to the first device, while “2” to the second. If one assumes a simple linear model of the moment of static characteristics of the devices, the dimensionless form of equation (1) can be rewritten as follows:

$$\ddot{x} + x - \mu\kappa_1\left(\ddot{\varphi}_1 \sin\varphi_1 + \dot{\varphi}_1^{\,2}\cos\varphi_1\right) - \mu\kappa_2\left(\ddot{\varphi}_2 \sin\varphi_2 + \dot{\varphi}_2^{\,2}\cos\varphi_2\right) = 0;$$

(2) $$\ddot{\varphi}_1 + a_1\dot{\varphi}_1 - b_1 - \mu\kappa_1 c_1 \ddot{x}\sin\varphi_1 = 0;$$

$$\ddot{\varphi}_2 + a_2\dot{\varphi}_2 - b_2 - \mu\kappa_2 c_2 \ddot{x}\sin\varphi_2 = 0,$$

where $\mu << 1$ appears as the-small parameter of the problem. The parameters κ_1 и κ_2 are of order of unity such that $\mu_1 = \mu\kappa_1$ and $\mu_2 = \mu\kappa_2$, where $\mu_1 = 2m_1 r_1 / M(m_1 + m_2)$ and $\mu_2 = 2m_2 r_2 / M(m_1 + m_2)$. We have introduced new notations: $a_i = \dfrac{k_{0i} + k_i}{I_1\omega_0}$, $b_i = \dfrac{M_i}{I_i\omega_0^2}$, $c_i = \dfrac{m(r_1 + r_2)}{4I_i}$ ($i = 1,2$). Here $\omega_0 = \sqrt{p/m}$ is the oscillation frequency of the platform in the absence of the engines, $x = 2\eta/(r_1 + r_2)$ is the new dimensionless linear coordinate measured in fractions of the radius of inertia of the eccentrics. The set (2), in contrast to the original equations, depends now on the dimensionless time $\tau = \omega_0 t$.

The problem (2) admits an effective study by the method of a small parameter. In order to explore this method, one should transform the system (2) to a standard form of the six equations resolved for the first derivatives. The intermediate steps of this procedure are as follows:

- introducing new variables y, ϖ_1, ϖ_2 associated with the initial dependent variables by differential relations

  $\frac{dx}{d\tau} = y$, $\frac{d\varphi_1}{d\tau} = \varpi_1$, $\frac{d\varphi_2}{d\tau} = \varpi_2$,

assuming that $\mu = 0$ in the set (2), one defines the transform to the new dependent variables based on the method of varied constants: $\varphi(\tau) = \tau + \alpha(\tau)$, $\varpi_1(\tau) = \nu_1(\tau)\exp(-a_1\tau)$, $\varpi_2(\tau) = \nu_2(\tau)\exp(-a_2\tau)$, $\varphi_1(\tau) = \Omega_1\tau + \beta_1(\tau)$, $\varphi_2(\tau) = \Omega_2\tau + \beta_2(\tau)$, where $x(\tau) = \rho(\tau)\sin(\varphi(\tau))$, $y(\tau) = \rho(\tau)\cos(\varphi(\tau))$, Ω_1 and Ω_2 are the partial angular velocities of devices. Here $\rho(\tau)$, $\alpha(\tau)$, $\nu_1(\tau)$, $\nu_2(\tau)$, $\beta_1(\tau)$, $\beta_2(\tau)$ are the six new variables of the problem. The notations of these new variables are the following: $\rho(\tau)$, $\alpha(\tau)$ is the amplitude and phase of oscillations of the base, respectively, $\nu_1(t)$, $\nu_2(t)$ are the angular accelerations and $\beta_1(\tau)$, $\beta_2(\tau)$ are the angular velocities of the rotors,

- the standard form suitable for further analysis corresponding to eqs.(2) is ready.

Because of the large record length of a standard form in itself is not given, but the interested reader can follow in detail the stages of its derivation in the algorithmic package Maple 12 on the site [9].

Solution of the system in a standard form is solved as series in the small parameter μ:

$$\rho(\tau) = \rho(T_1, T_2, \ldots) + \mu\rho^{(1)}(\tau) + \mu^2\rho^{(2)}(\tau) + \ldots;\ \alpha(\tau) = \alpha(T_1, T_2, \ldots) + \mu\alpha^{(1)}(\tau) + \mu^2\alpha^{(2)}(\tau) + \ldots;$$

(3) $$\nu_1(\tau) = \nu_1(T_1, T_2, \ldots) + \mu\nu_1^{(1)}(\tau) + \mu^2\nu_1^{(2)}(\tau) + \ldots;\ \nu_2(\tau) = \nu_2(T_1, T_2, \ldots) + \mu\nu_2^{(1)}(\tau) + \mu^2\nu_2^{(2)}(\tau) + \ldots;$$

$$\beta_1(\tau) = \beta_1(T_1, T_2, \ldots) + \mu\beta_1^{(1)}(\tau) + \mu^2\beta_1^{(2)}(\tau) + \ldots;$$

$$\beta_2(\tau) = \beta_2(T_1, T_2, \ldots) + \mu\beta_2^{(1)}(\tau) + \mu^2\beta_2^{(2)}(\tau) + \ldots.$$

Here, the kernel expansion depends upon the slow temporal scales $T_n = \mu^n\tau$, which characterize the evolution of resonant processes. The variables with superscripts denote small rapidly oscillating correction to the basic evolutionary solution.

Then it is necessary to identify the resonant conditions in the standard form. The resonance in the system (2) occurs within the first-order nonlinear approximation theory, when $\Omega_1 \sim 1$ and when $\Omega_2 \sim 1$ or if the both parameters are close to unity, $\Omega_2 \sim \Omega_1 \sim 1$. All these cases require a separate study. Now we are interested in the phenomenon of the phase synchronization in the system (2). This case, in particular, is realized when $\Omega_2 \sim \Omega_1$, though the both partial angular velocities should be sufficiently far from unity and less than unity, in order to overcome the instability predicted by the Sommefeld effect, since the first-order approximation resonance is in absent the system (2) in this case. Resonance is manifested in the second approximation.

In addition to the resonance associated with the standard phase synchronization in the system (2) there is one more resonance, when $2-\Omega_1-\Omega_2 \sim 0$, which apparently has no practical significance, since its angular velocities fall in the zone of instability, according to the Sommerfeld effect [].

Note that other resonances in the system (2) are absent within the second-order nonlinear approximation theory. The next section investigates these cases are in detail.

The synchronization. The matching condition $\Omega_2 \sim \Omega_1 \neq 1$.

After the substitution the expressions (3) into the standard form of equations and the separation of fast and slow motions within the first order approximation theory in small parameter μ one obtains the following information about the solution of the system. In the first approximation theory, the slow steady-state motions (when $t \to \infty$) are the same as in the linearised solution:

$$A = \text{const}, \ \alpha = \text{const}; \nu_1 = \text{const}, \ \nu_2 = \text{const}; B_1 = \text{const}; B_2 = \text{const}.$$

This means that the slowly varying generalized coordinates ρ, α, ν_1 and ν_2, β_1 and β_2 do not depend in the first approximation of any of the physical time τ nor the slow time T_1.

Solutions for the small non-resonant corrections appear as follows:

$$\varpi_1^{(1)}(\tau)=\frac{\mu c_1\kappa_1 A}{2\left(a_1^2+(1+\Omega_1)^2\right)\left(a_1^2+(1-\Omega_1)^2\right)}\left(\begin{array}{l}(\Omega_1-1)\left(a_1^2+(1+\Omega_1)^2\right)\sin\left((\Omega_1-1)\tau+\varphi_1-\alpha-\frac{\Omega_1}{a_1}\right)+\\ +a_1\left(a_1^2+(1+\Omega_1)^2\right)\cos\left((\Omega_1-1)\tau+\varphi_1-\alpha-\frac{\Omega_1}{a_1}\right)-\\ -\left((1+\Omega_1)\sin\left((\Omega_1+1)\tau+\varphi_1+\alpha-\frac{\Omega_1}{a_1}\right)\right)+\\ +a_1\left(a_1^2+(1-\Omega_1)^2\right)\cos\left((\Omega_1+1)\tau+\varphi_1+\alpha-\frac{\Omega_1}{a_1}\right)\end{array}\right)$$

$$\varpi_2^{(1)}(\tau)=\frac{\mu c_2\kappa_2 A}{2\left(a_2^2+(1+\Omega_2)^2\right)\left(a_2^2+(1-\Omega_2)^2\right)}\left(\begin{array}{l}(\Omega_2-1)\left(a_2^2+(1+\Omega_2)^2\right)\sin\left((\Omega_2-1)\tau+\varphi_2-\alpha-\frac{\Omega_2}{a_2}\right)+\\ +a_2\left(a_2^2+(1+\Omega_2)^2\right)\cos\left((\Omega_2-1)\tau+\varphi_2-\alpha-\frac{\Omega_2}{a_2}\right)-\\ -\left((1+\Omega_2)\sin\left((\Omega_2+1)\tau+\varphi_2+\alpha-\frac{\Omega_2}{a_2}\right)\right)+\\ +a_2\left(a_2^2+(1-\Omega_2)^2\right)\cos\left((\Omega_2+1)\tau+\varphi_2+\alpha-\frac{\Omega_2}{a_2}\right)\end{array}\right) \quad (4)$$

$$\rho^{(1)}(\tau)=-\frac{\mu}{2A}\left(\begin{array}{l}-\frac{\kappa_2\Omega_2^2\cos\left((\Omega_2-1)\tau+\varphi_2-\alpha-\frac{\Omega_2}{a_2}\right)}{\Omega_2-1}+\frac{\kappa_1\Omega_1^2\cos\left((\Omega_1+1)\tau+\varphi_1+\alpha-\frac{\Omega_1}{a_1}\right)}{\Omega_1+1}+\\ +\frac{\kappa_2\Omega_2^2\cos\left((\Omega_2+1)\tau+\varphi_2+\alpha-\frac{\Omega_2}{a_2}\right)}{\Omega_1+1}+\frac{\kappa_1\Omega_1^2\cos\left((\Omega_1-1)\tau+\varphi_1-\alpha-\frac{\Omega_1}{a_1}\right)}{\Omega_2-1}\end{array}\right)$$

$$\alpha^{(1)}(\tau)=-\frac{\mu}{2}\left(\begin{array}{l}\frac{\kappa_2\Omega_2^2\sin\left((\Omega_2-1)\tau+\varphi_2-\alpha-\frac{\Omega_2}{a_2}\right)}{\Omega_2-1}+\frac{\kappa_1\Omega_1^2\sin\left((\Omega_1+1)\tau+\varphi_1+\alpha-\frac{\Omega_1}{a_1}\right)}{\Omega_1+1}+\\ +\frac{\kappa_2\Omega_2^2\sin\left((\Omega_2+1)\tau+\varphi_2+\alpha-\frac{\Omega_2}{a_2}\right)}{\Omega_1+1}-\frac{\kappa_1\Omega_1^2\sin\left((\Omega_1-1)\tau+\varphi_1-\alpha-\frac{\Omega_1}{a_1}\right)}{\Omega_2-1}\end{array}\right)$$

$$\beta_1^{(1)}(\tau)=\frac{\mu c_1\kappa_1}{2a_1}\left(-\frac{\sin\left((\Omega_1-1)\tau+\varphi_1-\alpha-\frac{\Omega_1}{a_1}\right)}{\Omega_1-1}+\frac{\sin\left((\Omega_2+1)\tau+\varphi_2+\alpha-\frac{\Omega_2}{a_2}\right)}{\Omega_2+1}\right)$$

$$\beta_2^{(1)}(\tau)=\frac{\mu c_2\kappa_2}{2a_2}\left(-\frac{\sin\left((\Omega_2-1)\tau+\varphi_2-\alpha-\dfrac{\Omega_2}{a_2}\right)}{\Omega_2-1}+\frac{\sin\left((\Omega_1+1)\tau+\varphi_1+\alpha-\dfrac{\Omega_1}{a_1}\right)}{\Omega_1+1}\right).$$

This solution (4) describes a small perturbed motion of the base with the same frequencies as the angular velocities of rotation of drivers, which is manifested in the appearance of combination frequencies in the expression for the corrections to the amplitude $\rho^{(1)}(\tau)$ and the phase $\alpha^{(1)}(t)$. Amendments to the angular accelerations $\varpi_1^{(1)}(t)$, $\varpi_2^{(1)}(t)$ and the velocities $\beta_1^{(1)}(\tau)$, $\beta_2^{(1)}(\tau)$ also contain the similar small-amplitude combination harmonics at the difference and sum.

Now the solution of the first-order approximation is constructed. This decision has not suitable for describing the synchronization effect and call to continue further manipulations with the equations along the small-parameter method. Using the solution (4), after the substitution in eqs.(3), one obtains the desired equation of the second-order nonlinear approximation, describing the synchronization phenomenon of a pair of drivers on an elastic foundation.

So that, after the second substitution of the modified representation (3) in the standard form and the separation of motions into slow and fast ones, one obtains the following evolution equations.

$$\frac{d\rho}{dT_2}=(S-D)\rho-P(\Omega_1-\Omega_2)\frac{\sin\Psi}{\rho}$$

(5)

$$\frac{d\Psi}{dT_2}=\delta+Q\sin\Psi+R\rho^2,$$

where $\Psi(T_2)=\varphi_1(T_2)-\varphi_2(T_2)-\Delta T_2+\dfrac{\Omega_2}{a_2}-\dfrac{\Omega_1}{a_1}$ is the new slow variable ($\Delta=\Omega_1-\Omega_2$), and $\delta=\dfrac{\Omega_1-\Omega_2}{\mu^2}$ denotes the small detuning of the partial angular velocities. The coefficients of equations (5) appear as follows:

$$S=\frac{a_1c_1\kappa_1^2\left(3\Omega_1^2+a_1^2+1\right)}{4(\Omega_2^2-1)\left(a_2^2+(1-\Omega_2)^2\right)\left(a_2^2+(1+\Omega_2)^2\right)}+\frac{a_2c_2\kappa_2^2\left(3\Omega_2^2+a_2^2+1\right)}{4(\Omega_1^2-1)\left(a_1^2+(1-\Omega_1)^2\right)\left(a_1^2+(1+\Omega_1)^2\right)};$$

$$P = \frac{\kappa_1 \kappa_2 \Omega_1^2 \Omega_2^2 (1 + \Omega_1 \Omega_2)}{4(1-\Omega_1^2)(1-\Omega_2^2)};$$

$$Q = \frac{\kappa_1 \kappa_2}{2}\left[\frac{c_1}{a_1}\left(\frac{\Omega_2^2(\Omega_2^2-2)}{1-\Omega_2^2}\right) + \frac{c_2}{a_2}\left(\frac{\Omega_1^2(\Omega_1^2-2)}{1-\Omega_1^2}\right)\right];$$

$$R = \frac{c_2^2 \kappa_2^2 \Omega_2 (\Omega_2^2 + a_2^2 + 3)}{4(\Omega_2^2 - 1)(a_2^2 + (1-\Omega_2)^2)(a_2^2 + (1+\Omega_2)^2)} - \frac{c_1^2 \kappa_1^2 \Omega_1 (\Omega_1^2 + a_1^2 + 3)}{4(\Omega_1^2 - 1)(a_1^2 + (1-\Omega_1)^2)(a_1^2 + (1+\Omega_1)^2)}.$$

Let the detuning be zero, then these equations are highly simplified up to the full their separation:

$$\frac{d\rho}{dT_2} = (S - D)\rho$$

(6)

$$\frac{d\Psi}{dT_2} = Q \sin\Psi + R\rho^2 .$$

Equations (5) (5) represent a generalization of the standard basic equations of the theory of phase synchronization [10], whose structure reads as follows

(7) $$\frac{d\Psi}{dT_2} = \Delta + Q \sin\Psi .$$

Formally, this equation follows from the generalized model (5) or (6), if we put $\rho = 0$. The equation (7) has the general solution

$$\psi(T_2) = 2\arctan\left(\frac{1}{\Delta}\left(\tan\left(\frac{T_2}{2}\sqrt{\Delta^2 - Q^2} + \frac{C}{2}\sqrt{\Delta^2 - Q^2}\right)\sqrt{\Delta^2 - Q^2} - Q\right)\right),$$

where C is an arbitrary constant of integration. This solution implies the criterion of the stable phase synchronization:

(8) $$\Delta^2 - Q^2 < 0,$$

which indicates that the occurrence of the stable synchronization the phase detuning must be small enough, compared with the phase modulation parameter. If this condition is not observed, then the system leaves the zone of synchronization.

On the other hand the refined model (6) says that for the stable synchronization the performance of the above conditions (8) is not enough. It is also necessary condition that the coefficient of the resonant excitation of vibrations in the base S should not exceed the rate of energy dissipation D, i.e. $S < D$. The last restriction significantly alters the stability zone of synchronization in the parameter space of the system that, in fact, is demonstrated below on the specific computational examples.

Examples of stable and unstable regimes of synchronization

The table below shows the calculation of the different theoretical implementations of stable and unstable regimes of the phase synchronization. The example 1 (see the first line in the table) demonstrates is a robust synchronization with a small mismatch between the angular velocities of drivers, i.e. $\delta = 0.1$. The example 2 (see, respectively, the second line in the table, etc.) displays an unstable phase-synchronization regime at the small difference between the angular velocities, i.e. $\delta = 0.1$. Reach a steady state of synchronization in this example by adding a damping element with damping coefficient $\eta \geq 0.009$. The example number 3. This is the robust synchronization for the small differences in eccentrics ($\kappa_1 - \kappa_2 = 0.2$) and equal angular velocities. The example number 4. This is an unstable synchronization mode with the small differences in eccentrics ($\kappa_1 - \kappa_2 = 0.2$) and the small mismatch in angular velocities, i.e. $\delta = 0.1$. One can reach a steady state of synchronization in this example by adding a dissipative element with the damping coefficient $\eta \geq 0.009$. The example number 5. This is an unstable synchronization regime with the small differences in eccentrics. One cannot reach any stable synchronization regime in this example, it is impossible, even when adding any damping element. The example number 6. This is an unstable regime of synchronization at the different angular speeds. It is also impossible to achieve any sustainable sync mode in this case.

	μ	c_1	c_2	κ_1	κ_2	a_1	a_2	Ω_1	Ω_2	$\Delta^2 - Q^2$	S
1	0.1	1	1	0.5	0.5	1	1	0.751	0.75	-0.244	-0.204
2	0.1	1	1	0.5	0.5	1	1	0.251	0.25	-0.072	0.008
3	0.1	1	1	0.6	0.4	1	1	0.25	0.25	-0.075	-0.001
4	0.1	1	1	0.6	0.4	1	1	0.251	0.25	-0.075	0.009
5	0.1	1	1	0.6	0.4	1	1	1.25	1.25	0.239	-0.085

6	0.1	1	1	0.5	0.5	1	1	0.26	0.25	0.998	-0.007

Table. Parameters of stable and unstable regimes of synchronization.

The matching condition $2-\Omega_1-\Omega_2 \sim 0$.

After substitution from the expressions (3) into the standard form of equations (2), separation of fast and slow motions in the first-order approximation in the small parameter μ, under the assumption that $2-\Omega_1-\Omega_2 \sim 0$ one obtains the following evolutionary equations

$$\frac{d\rho}{dT_2} = (S-D)\rho - P(2-\Omega_1-\Omega_2)\frac{\sin\Psi}{4\rho};$$

(9)

$$\frac{d\Psi}{dT_2} = Q - P(2-\Omega_1-\Omega_2)\frac{\cos\Psi}{\rho^2} - R\rho^2 + \delta,$$

where $\Psi(T_2) = \varphi_1(T_2) + \varphi_2(T_2) - 2\alpha(T_2) - \Delta T_2 - \frac{\Omega_1}{a_1} - \frac{\Omega_2}{a_2}$ is the new slow variable ($\Delta = 2-\Omega_1-\Omega_2$), and $\delta = \frac{2-\Omega_1-\Omega_2}{\mu^2}$ being the small detuning of partial angular velocities. The coefficients of equations (9) are as follows:

$$S = \frac{a_1 c_1 \kappa_1^2 (3\Omega_1^2 + a_1^2 + 1)}{4(1-\Omega_1)^2 (a_1^2 + (1-\Omega_1)^2)(a_1^2 + (1+\Omega_1)^2)} + \frac{a_2 c_2 \kappa_2^2 (3\Omega_2^2 + a_2^2 + 1)}{4(1-\Omega_3)^2 (a_2^2 + (1-\Omega_2)^2)(a_2^2 + (1+\Omega_2)^2)};$$

$$P = \frac{\kappa_1 \kappa_2 \Omega_1^2 \Omega_2^2}{2(1-\Omega_1)(1-\Omega_2)};$$

$$Q = \frac{\left(\Omega_1^4 + \left(2a_1^2 - \frac{5}{2}\right)\Omega_1^2 + a_1^4 + \frac{3}{2}a_1^2 + \frac{1}{2}\right)c_1\kappa_1^2}{(a_1^2 + (1-\Omega_1)^2)(a_1^2 + (1+\Omega_1)^2)} + \frac{\left(\Omega_2^4 + \left(2a_2^2 - \frac{5}{2}\right)\Omega_2^2 + a_2^4 + \frac{3}{2}a_2^2 + \frac{1}{2}\right)c_2\kappa_2^2}{(a_2^2 + (1-\Omega_2)^2)(a_2^2 + (1+\Omega_2)^2)};$$

$$R = \frac{c_1^2 \kappa_1^2 \Omega_1 (\Omega_1^2 + a_1^2 + 3)}{4(1-\Omega_1)^2 (a_1^2 + (1-\Omega_1)^2)(a_1^2 + (1+\Omega_1)^2)} + \frac{c_2^2 \kappa_2^2 \Omega_2 (\Omega_2^2 + a_2^2 + 3)}{4(1-\Omega_3)^2 (a_2^2 + (1-\Omega_2)^2)(a_2^2 + (1+\Omega_2)^2)}.$$

The resonance of this type when $2-\Omega_1-\Omega_2 \sim 0$, as already mentioned, has no practical significance, since for its implementation the angular speed of one driver, say, for example, the second, i.e. $\Omega_2 > 1$, should fall in the zone of instability, according to the Sommerfeld effect.

Let the detuning be zero, then these equations are highly simplified up to the full their separation:

$$\frac{d\rho}{dT_2} = (S - D)\rho;$$

(10)

$$\frac{d\Psi}{dT_2} = Q - R\rho^2.$$

The formal criterion of stability is extremely simple. Namely, the coefficient of the resonant excitation of vibrations in the base S exceeds no the rate of energy dissipation, D, i.e. $S < D$.

Conclusions

Synchronous rotations of drivers are almost idle and required no a high-powered energy set in this case [11]. Most responsible treatment for the drivers is their start, i.e. a transition from the rest to a steady-state rotation. Using some vibration absorbers for high-powered electromechanical systems has advantageous for the two main reasons. On the one hand it provides a tool for substantially mitigating the effects of transient shocking loads during the time of growth the acceleration of drivers. This contributes to preservations of the electromechanical system and save energy. On the other hand the there is an ability to configure the appropriate damping properties of vibration absorbers to create a stable regime of synchronization when it is profitable, or even get rid of him, to destroy the synchronous movement, creating conditions for a dynamic interchange drivers.

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